POWER SYSTEMS-III

(R20- R20A0209)

LECTURE NOTES

B. TECH

(III YEAR – II SEM)

(2023-2024)

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MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY
(Autonomous Institution – UGC, Govt. of India)
(Affiliated to JNTU, Hyderabad, Approved by AICTE - Accredited by NBA &NAAC – ‘A’ Grade - ISO 9001:2015 Certified)

MALLA REDDY COLLEGE OF ENGINEERING AND TECHNOLOGY

III YEAR B. Tech EEE– II SEM **L/T/P/C**

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(R20A0209) POWER SYSTEMS - III

COURSE OBJECTIVES:

- To understand and develop Y bus matrices
- To give the knowledge on per unit system.
- To understand and develop Z bus matrices
- To give the knowledge on faults analysis.
- Give the knowledge of iterative method in power systems.
- To understand the concepts load flow studies.

UNIT I:

PER UNIT REPRESENTATION OF POWER SYSTEMS: The one-line diagram, impedance and reactance diagrams, per unit quantities, changing the base of per unit quantities, advantages of per unit system.

POWER SYSTEM NETWORK MATRICES: Bus Incidence Matrix, Y-bus formation by Direct andSingular Transformation Methods, Numerical Problems.

UNIT II:

FORMATION OF Z-BUS: Partial network, Algorithm for the Modification of Z Bus Matrix for addition element for the following cases: Addition of element from a new bus to reference, Addition of element from a new bus to an old bus, Addition of element between an old bus to reference and addition of element between two old buses

UNIT–III

SYMMETRICAL COMPONENTS AND FAULT CALCULATIONS: Significance of positive, negative and zero sequence components, sequence impedances and sequence networks, fault calculations, sequence network equations, single line to ground fault, line to line fault, double line to ground fault, three phase faults, faults with fault impedance.

UNIT–IV

LOAD FLOW STUDIES I: Derivation of Static load flow equations. Load Flow Solutions Using Gauss Seidel Method: Acceleration Factor, Load flow solution with and without P-V buses, Algorithm and Flowchart. Numerical Load flow Solution for Simple Power Systems (Max. 3-Buses): Determination of Bus Voltages, Injected Active and Reactive Powers (Sample OneIteration only) and finding Line Flows/Losses for the given Bus Voltages.

UNIT–V

LOAD FLOW STUDIES II: Numerical Load flow Solution for Simple Power Systems (Max. 3-Buses): Determination of Bus Voltages, Injected Active and Reactive Powers (One Iteration only) and finding Line Flows/Losses for the given Bus Voltages, Newton Raphson Method (Polar coordinates only): Load Flow Solution with and without P-V Buses, Derivation of Jacobian Elements, Fast Decoupled Method**.**

TEXT BOOKS:

1. C.L.Wadhwa, Electrical Power Systems, 3rd Edn, New Age International Publishing Co.,2001.
2. D.P.Kothari and I.J.Nagrath, Modern Power System Analysis, 4th Edn, Tata McGraw HillEducation Private Limited2011.

REFERENCEBOOKS:

1. D. P. Kothari: Modern Power System Analysis-Tata McGraw Hill Pub. Co. 2003
2. Hadi Scadat: Power System Analysis – Tata McGraw Hill Pub. Co.2002
3. W.D. Stevenson: Elements of Power system Analysis – McGraw Hill International StudentEdition.

COURSE OUTCOMES:

At the end of the course the student will be able to:

- Understand the concept of per unit system and faults in power systems.
- Evaluate the admittance matrix of a given power systems.
- Analyze the power system using iterative methods.
- Understand the concept of load flow studies in power system.
- Understand the PF and computer control in power system.

UNIT-I

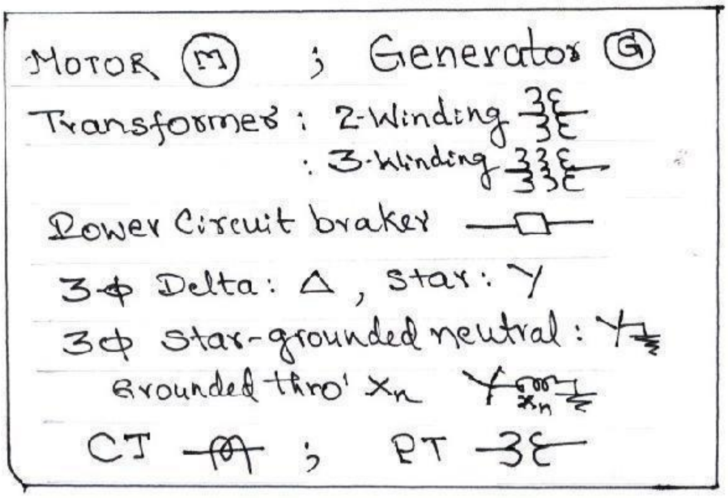
PER UNIT REPRESENTATION OF POWER SYSTEMS

One Line Diagram

In practice, electric power systems are very complex and their size is unwieldy. Itis very difficult to represent all the components of the system on a single frame. The complexities could be in terms of various types of protective devices, machines (transformers, generators, motors, etc.), their connections (star, delta, etc.), etc. Hence, for the purpose of power system analysis, a simple single phase equivalent circuit is developed called, the one line diagram (OLD) or the single line diagram (SLD). *An SLD is thus, the concise form of representing a given power system.* It is to be noted that agiven SLD will contain only such data that are relevant to the system analysis/study under consideration. For example, the details of protective devices need not be shown for load flow analysis nor it is necessary to show the details of shunt values for stability studies.

Symbols used for SLD

Various symbols are used to represent the different parameters and machines as single phase equivalents on the SLD,. Some of the important symbols used are as listed in the table of Figure 1.

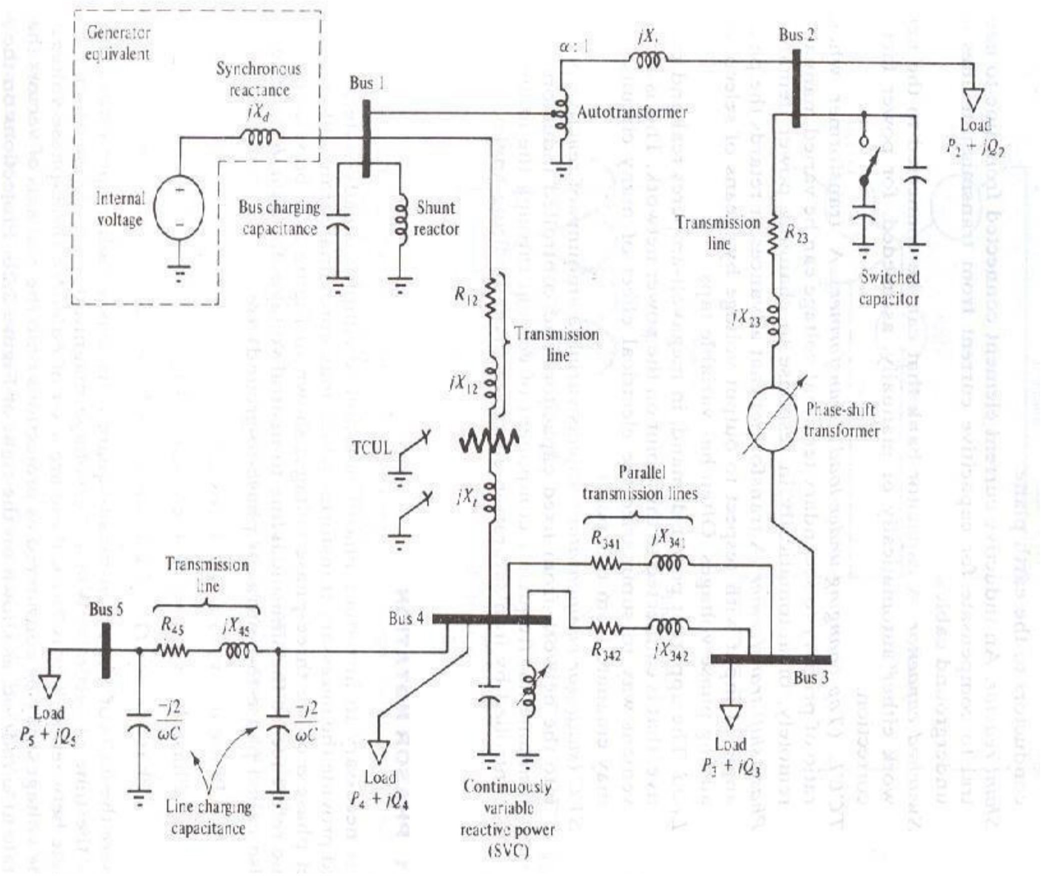


MOTOR Ⓜ ; Generator Ⓖ
Transformer : 2-Winding
: 3-Winding
Power Circuit braker —□—
3-ϕ Delta : Δ , Star : Y
3ϕ Star-grounded neutral :
Grounded thro' x_n x_n
CT ; PT

Figure 1. TABLE OF SYMBOLS FOR USE ON SLDS

Example system

Consider for illustration purpose, a sample example power system and data as under:
Generator 1: 30 MVA, 10.5 KV, X”= 1.6 ohms, Generator 2: 15 MVA, 6.6 KV, X”=
ohms, Generator 3: 25 MVA, 6.6 KV, X”= 0.56 ohms, Transformer 1 (3-phase): 15 MVA, 33/11 KV, X=15.2 ohms/phase on HT side, Transformer 2 (3-phase): 15 MVA, 33/6.2 KV, X=16.0 ohms/phase on HT side, Transmission Line: 20.5 ohms per phase, Load A: 15 MW, 11 KV, 0.9 PF (lag); and Load B: 40 MW, 6.6 KV, 0.85 PF (lag). The corresponding SLD incorporating the standard symbols can be shown as in figure 2.



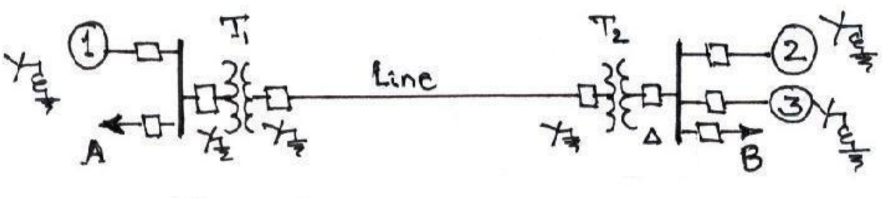


Figure 2. SAMPLE SYSTEM OLD

It is observed here, that the generators are specified in 3-phase MVA, L-L voltage and per phase Y-equivalent impedance, transformers are specified in 3- phase MVA, L-L voltage transformation ratio and per phase Y-equivalent impedance on any one side and the loads are specified in 3-phase MW, L-L voltage and power factor.

Impedance Diagram

The impedance diagram on single-phase basis for use under balanced conditions can be easily drawn from the SLD. The following assumptions are made in obtaining the impedance diagrams.

Assumptions:

1. The single phase transformer equivalents are shown as ideals with impedances on appropriate side (LV/HV),
2. The magnetizing reactances of transformers are negligible,
3. The generators are represented as constant voltage sources with series resistance or reactance,
4. The transmission lines are approximated by their equivalent π-Models,
5. The loads are assumed to be passive and are represented by a series branch of resistance or reactance and
6. Since the balanced conditions are assumed, the neutral grounding impedances do not appear in the impedance diagram.

Example system

As per the list of assumptions as above and with reference to the system of figure 2, the impedance diagram can be obtained as shown in figure 3.

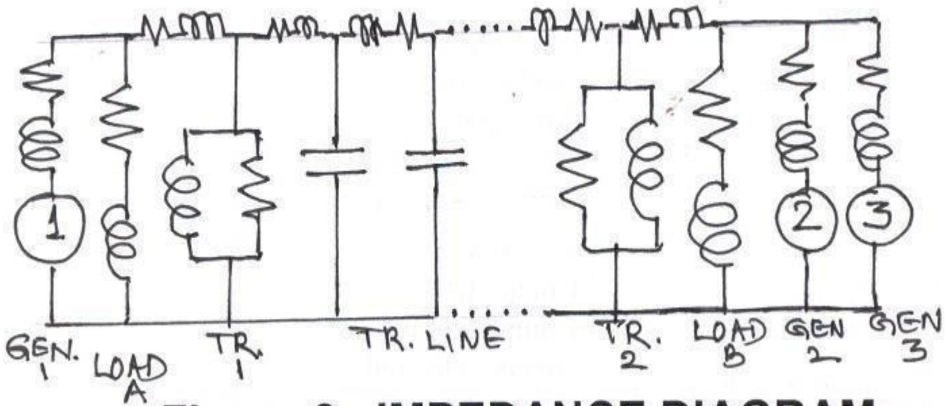


Figure 3. IMPEDANCE DIAGRAM

Reactance Diagram

With some more additional and simplifying assumptions, the impedance diagram can be simplified further to obtain the corresponding reactance diagram. The following are the assumptions made.

Additional assumptions:

- The resistance is often omitted during the fault analysis. This causes a very negligible error since, resistances are negligible
- Loads are Omitted
- Transmission line capacitances are ineffective &
- Magnetizing currents of transformers are neglected.

Example system

as per the assumptions given above and with reference to the system of figure 2 and figure 3, the reactance diagram can be obtained as shown in figure 4.

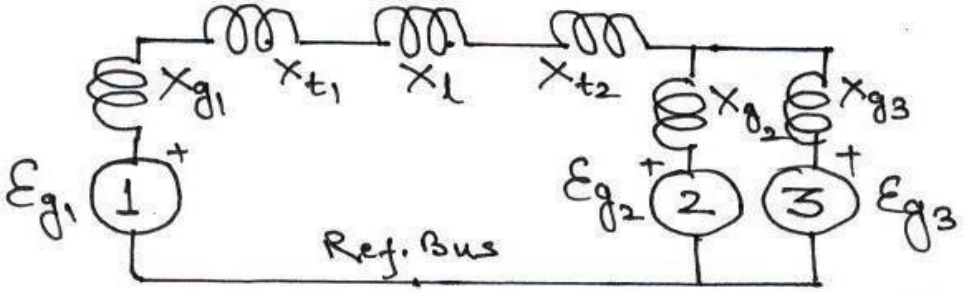


Figure 4. REACTANCE DIAGRAM

Note: These impedance & reactance diagrams are also refered as the *Positive Sequence Diagrams/ Networks.*

Per Unit Quantities

during the power system analysis, it is a usual practice to represent current, voltage, impedance, power, etc., of an electric power system in per unit or percentage of the base or reference value of the respective quantities. The numerical per unit (pu) value of any quantity is its ratio to a chosen base value of the same dimension. Thus a pu value is a normalized quantity with respect to the chosen base value.

Definition: Per Unit value of a given q*uantity is the ratio of the **actual value** in any given unit to the **base value** in the same unit.* The percent value is 100 times the pu value. Both the pu and percentage methods are simpler than the use of actual values. Further, the main advantage in using the pu system of computations is that the result that comes out of the sum, product, quotient, etc. of two or more pu values is expressed in perunit itself.

In an electrical power system, the parameters of interest include the current, voltage, complex power (VA), impedance and the phase angle. Of these, the phase angle is dimensionless and the other four quantities can be described by knowing any two of them. Thus clearly, an arbitrary choice of any two base values will evidently fix the other base values.

Normally the nominal voltage of lines and equipment is known along with the complex power rating in MVA. Hence, in practice, the base values are chosen for complex power (MVA) and line voltage (KV). The chosen base MVA is the same for all the parts of the system. However, the base voltage is chosen with reference to a particular section of the system and the other base voltages (with reference to the other sections of the systems, these sections caused by the presence of the transformers) are then related to the chosen one by the turns- ratio of the connecting transformer.

If I_b is the base current in kilo amperes and V_b, the base voltage in kilovolts, then the base MVA is, $S_b = (V_b I_b)$. Then the base values of current & impedance are given by

$$\text{Base current (kA), } I_b = MVA_b/KV_b$$
$$= S_b/V_b \qquad (1.1)$$

$$\text{Base impedance, } Zb = (V_b/I_b)$$
$$= (KV_b^2 / MVA_b) \qquad (1.2)$$

Hence the per unit impedance is given by

$$Z_{pu} = Z_{ohms}/Z_b$$
$$= Z_{ohms} (MVA_b/KV_b^2) \qquad (1.3)$$

In 3-phase systems, KV_b is the line-to-line value & MVA_b is the 3-phase MVA. [1-phase MVA = (1/3) 3-phase MVA].

Changing the base of a given pu value:

It is observed from equation (3) that the pu value of impedance is proportional directly to the base MVA and inversely to the square of the base KV. If Z_{pu}^{new} is the pu impedance required to be calculated on a new set of base values: MVA_b^{new} & KV_b^{new} from the already given per unit impedance Z_{pu}^{old}, specified on the old set of base values, MVA_b^{old} & KV_b^{old} , then we have

$$Z_{pu}^{new} = Z_{pu}^{old} (MVA_b^{new}/MVA_b^{old}) (KV_b^{old}/KV_b^{new})^2 \qquad (1.4)$$

On the other hand, the change of base can also be done by first converting the given pu impedance to its ohmic value and then calculating its pu value on the new set of base values.

Merits and Demerits of pu System

Following are the advantages and disadvantages of adopting the pu system of computations in electric power systems:

Merits

- The pu value is the same for both 1-phase and & 3-phase systems
- The pu value once expressed on a proper base, will be the same when refereed to either side of the transformer. Thus the presence of transformer is totally eliminated
- The variation of values is in a smaller range 9nearby unity). Hence the errors involved in pu computations are very less.
- Usually the nameplate ratings will be marked in pu on the base of the name plate ratings, etc.

Demerits:

- If proper bases are not chosen, then the resulting pu values may be highly absurd (such as 5.8 pu, -18.9 pu, etc.). This may cause confusion to the user. However, this problem can be avoided by selecting the base MVA near the high-rated equipment and a convenient base KV in any section of the system.

pu Impedance / Reactance Diagram

for a given power system with all its data with regard to the generators, transformers, transmission lines, loads, etc., it is possible to obtain the corresponding impedance or reactance diagram as explained above. If the parametric values are shown in pu on the properly selected base values of the system, then the diagram is refered as the per unit impedance or reactance diagram. In forming a pu diagram, the following are the procedural steps involved:

1. Obtain the one line diagram based on the given data
2. Choose a common base MVA for the system
3. Choose a base KV in any one section (Sections formed by transformers)
4. Find the base KV of all the sections present
5. Find pu values of all the parameters: R,X, Z, E, etc.
6. Draw the pu impedance/ reactance diagram.

POWER SYSTEM NETWORK MATRICES

1. FORMATION OF Y_{BUS} AND Z_{BUS}

The bus admittance matrix, YBUS plays a very important role in computer aided power system analysis. It can be formed in practice by either of the methods as under:

1. Rule of Inspection
2. Singular Transformation
3. Non-Singular Transformation
4. ZBUS Building Algorithms, etc.

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

Frames of Reference:

Bus Frame of Reference: There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

EBUS = ZBUS IBUS
IBUS = YBUS EBUS

Branch Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

EBR = ZBR IBR
IBR = YBR EBR

Loop Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

ELOOP = ZLOOP ILOOP
ILOOP = YLOOP ELOOP

Of the various network matrices refered above, the bus admittance matrix (YBUS) and the bus impedance matrix (ZBUS) are determined for a given power system by the rule of inspection as explained next.

Rule of Inspection

Consider the 3-node admittance network as shown in figure5. Using the basic branch relation: I = (YV), for all the elemental currents and applying Kirchhoff"s Current Law principle at the nodal points, we get the relations as under:

At node 1: I1 =Y1V1 + Y3 (V1-V3) + Y6 (V1 – V2)
At node 2: I2 =Y2V2 + Y5 (V2-V3) + Y6 (V2 – V1)

At node 3: 0 = Y3 (V3-V1) + Y4V3 + Y5 (V3 – V2) **(12)**

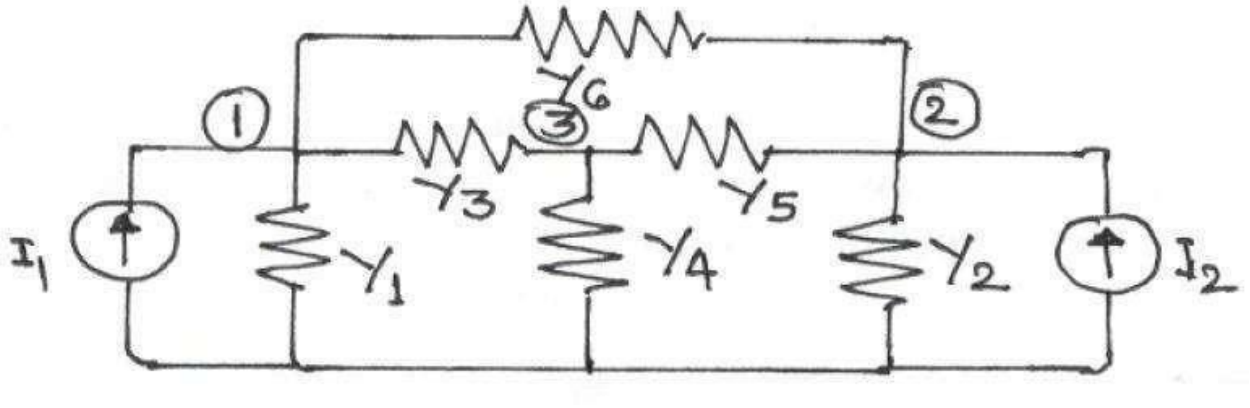


Fig. 3 Example System for finding YBUS

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} \qquad (13)$$

In other words, the relation of equation (9) can be represented in the form
IBUS = YBUS EBUS **(14)**
Where, YBUS is the bus admittance matrix, IBUS & EBUS are the bus current and bus voltage vectors respectively. By observing the elements of the bus admittance matrix, YBUS of equation (13), it is observed that the matrix elements can as well be obtained bya simple inspection of the given system diagram:
Diagonal elements: A diagonal element (Yii) of the bus admittance matrix, YBUS, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,
Off Diagonal elements: An off-diagonal element (Yij) of the bus admittance matrix,YBUS, is equal to the negative of the admittance value of the connecting element present between the buses I and j, if any. This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

Yii = S yij (j = 1,2,…….n)
Yij = - yij (j = 1,2,…….n) **(15)**

For i = 1,2,….n, n = no. of buses of the given system, yij is the admittance of element connected between buses i and j and yii is the admittance of element connected between bus i and ground (reference bus).

Bus impedance matrix

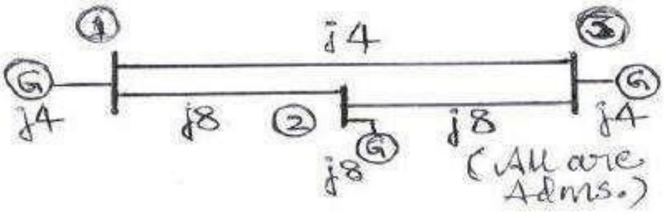
In cases where, the bus impedance matrix is also required, it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are interinvertible.

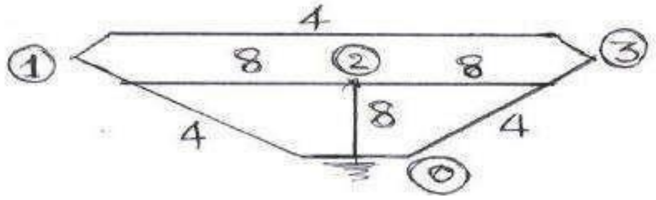
Note: It is to be noted that the rule of inspection can be applied only to those power systems that do not have any mutually coupled elements.

Examples on Rule of Inspection:

Example 6: Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection

$$Y_{BUS} = j\begin{vmatrix} 16 & -8 & -4 \\ -8 & 24 & -8 \\ -4 & -8 & 16 \end{vmatrix}$$

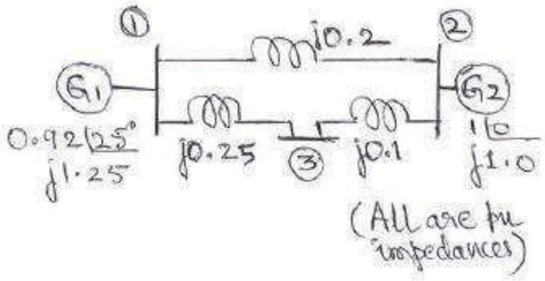


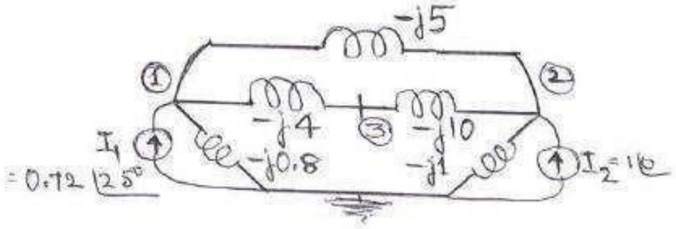


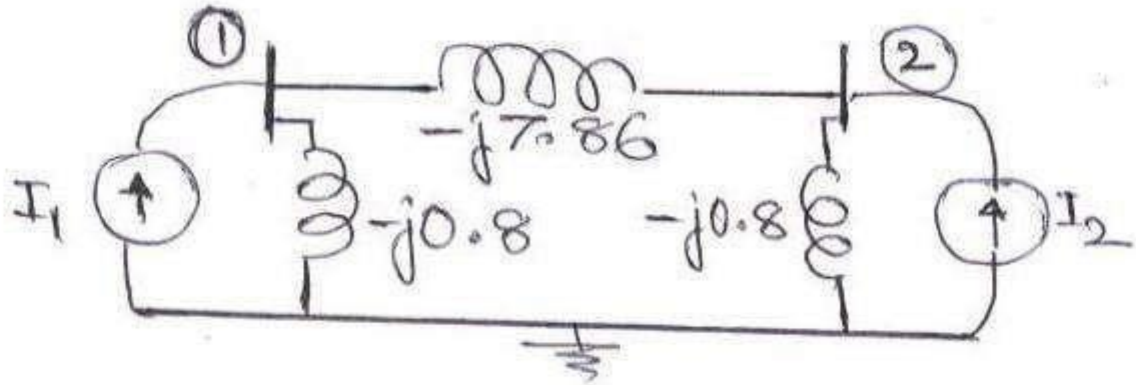
Example 7: Obtain YBUS for the impedance network shown aside by the rule of inspection. Also, determine YBUS for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.

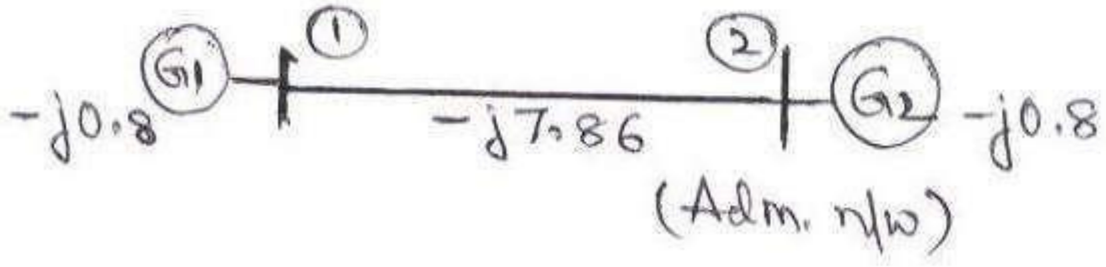
$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1}$$









$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$$Y_{BUS} = j\begin{vmatrix} -8.66 & 7.86 \\ 7.86 & -8.66 \end{vmatrix}$$

SINGULAR TRANSFORMATIONS

The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements. However, they do not contain any information about the behaviour of the interconnected network variables. Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

Bus admittance matrix, YBUS and Bus impedance matrix, ZBUS

In the bus frame of reference, the performance of the interconnected network is described by *n* independent nodal equations, where *n* is the total number of buses (n+1nodes are present, out of which one of them is designated as the reference node).
For example a 5-bus system will have 5 external buses and 1 ground/ ref. bus). The performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

IBUS = YBUS EBUS **(17)**
Where EBUS = vector of bus voltages measured with respect to reference bus
IBUS = Vector of currents injected into the bus
YBUS = bus admittance matrix
The performance equation of the primitive network in admittance form is given by
i + j = [y] v
Pre-multiplying by At (transpose of A), we obtain

At i +At j = At [y] v **(18)**
However, as per equation (4),

At i =0,
since it indicates a vector whose elements are the algebraic sum of element currents incident at a bus, which by Kirchhoff‟s law is zero. Similarly, At j gives the algebraic sum of all source currents incident at each bus and this is nothing but the total current injected at the bus. Hence,

At j = IBUS **(19)**
Thus from (18) we have, IBUS = At [y] v **(20)**
However, from (5), we have
v =A EBUS
And hence substituting in (20) we get,

IBUS = At [y] A EBUS **(21)**
Comparing (21) with (17) we obtain,

YBUS = At [y] A **(22)**
The bus incidence matrix is rectangular and hence singular. Hence, (22) gives a singular transformation of the primitive admittance matrix [y]. The bus impedance matrix is given by ,

ZBUS = YBUS^{-1} **(23)**
Note: This transformation can be derived using the concept of power invariance, however, since the transformations are based purely on KCL and KVL, the transformation will obviously be power invariant.

Examples on Singular Transformation:

Example 8: For the network of Fig E8, form the primitive matrices [z] & [y] and obtain the bus admittance matrix by singular transformation. Choose a Tree T(1,2,3). The data is given in Table E8.

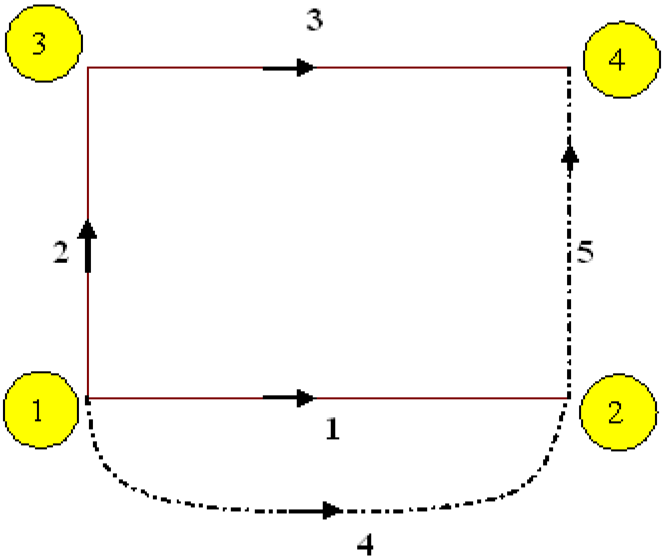
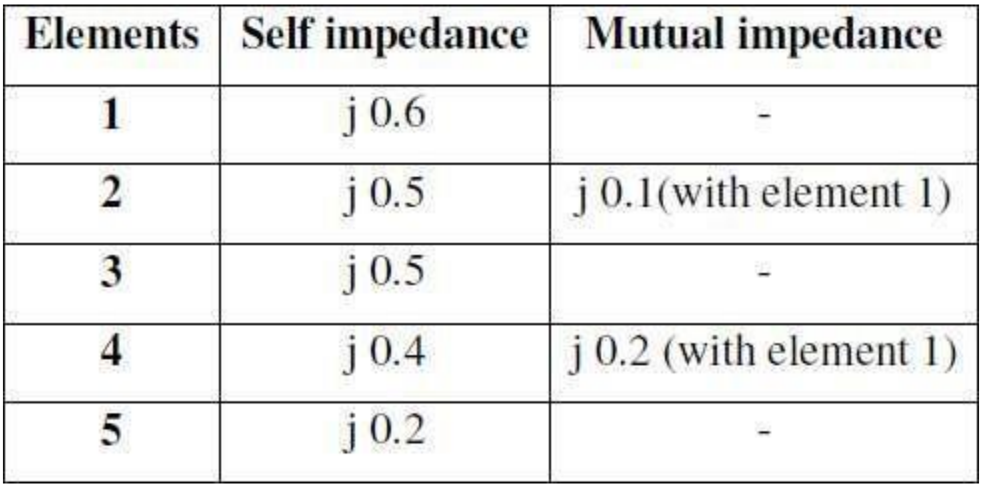


Fig E8 System for Example-8

Table E8: Data for Example-8



Elements	Self impedance	Mutual impedance
1	j 0.6	-
2	j 0.5	j 0.1(with element 1)
3	j 0.5	-
4	j 0.4	j 0.2 (with element 1)
5	j 0.2	-

Solution:

The bus incidence matrix is formed taking node 1 as the reference bus.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

The primitive incidence matrix is given by

$$[z]= \begin{bmatrix} j0.6 & j0.1 & 0.0 & j0.2 & 0.0 \\ j0.1 & j0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & j0.5 & 0.0 & 0.0 \\ j0.2 & 0.0 & 0.0 & j0.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & j0.2 \end{bmatrix}$$

The primitive admittance matrix [y] = [z]-1 and given by,

$$[y]= \begin{bmatrix} -j2.0833 & j0.4167 & 0.0 & j1.0417 & 0.0 \\ j0.4167 & -j2.0833 & 0.0 & -j0.2083 & 0.0 \\ 0.0 & 0.0 & -j2.0 & 0.0 & 0.0 \\ j1.0417 & -j0.2083 & 0.0 & -j3.0208 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -j5.0 \end{bmatrix}$$

The bus admittance matrix by singular transformation is obtained as

$$Y_{BUS} = A^{t}\,[y]\,A = \begin{bmatrix} -j8.0208 & j0.2083 & j5.0 \\ j0.2083 & -j4.0833 & j2.0 \\ j5.0 & j2.0 & -j7.0 \end{bmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1} = \begin{bmatrix} j0.2713 & j0.1264 & j0.2299 \\ j0.1264 & j0.3437 & j0.1885 \\ j0.2299 & j0.1885 & j0.3609 \end{bmatrix}$$

SUMMARY

The formulation of the mathematical model is the first step in obtaining the solution of any electrical network. The independent variables can be either currents or voltages. Correspondingly, the elements of the coefficient matrix will be impedances oradmittances.

Network equations can be formulated for solution of the network using graph theory, independent of the nature of elements. In the graph of a network, the tree-branches and links are distinctly identified. The complete information about the interconnection of the network, with the directions of the currents is contained in the bus incidence matrix.

The information on the nature of the elements which form the interconnected network is contained in the primitive impedance matrix. A primitive element can be represented in impedance form or admittance form. In the bus frame of reference, the performance of the interconnected system is described by *(n-1)* nodal equations, where *n* is the number of nodes. The bus admittance matrix and the bus impedance matrix relate the bus voltages and currents. These matrices can be obtained from the primitive impedance and admittance matrices.

FORMATION OF BUS IMPEDANCE MATRIX

NODE ELIMINATION BY MATRIX ALGEBRA

Nodes can be eliminated by the matrix manipulation of the standard node equations. However, *only those nodes at which current does not enter or leave the network can be considered for such elimination*. Such nodes can be eliminated either in one group or by taking the eligible nodes one after the other for elimination, as discussed next.

CASE-A: Simultaneous Elimination of Nodes:

Consider the performance equation of the given network in bus frame of reference in admittance form for a n-bus system, given by:

IBUS = YBUS EBUS **(1)**

Where IBUS and EBUS are n-vectors of injected bus current and bus voltages and YBUS is the square, symmetric, coefficient bus admittance matrix of order n. Now, of the n buses present in the system, let p buses be considered for node elimination so that the reduced system after elimination of p nodes would be retained with m (= n-p) nodes only. Hence the corresponding performance equation would be similar to (1) except that the coefficient matrix would be of order m now, i.e.,

IBUS = YBUS^{new} EBUS **(2)**

Where YBUS^{new} is the bus admittance matrix of the reduced network and the vectors

IBUS and EBUS are of order m. It is assumed in (1) that IBUS and EBUS are obtained with their elements arranged such that the elements associated with p nodes to be eliminated are in the lower portion of the vectors. Then the elements of YBUS also get located accordingly so that (1) after matrix partitioning yields,

$$\begin{vmatrix} I_{BUS\text{-}m} \\ I_{BUS\text{-}p} \end{vmatrix} = \begin{array}{c|cc|} & m & p \\ m & Y_A & Y_B \\ p & Y_C & Y_D \end{array} \begin{vmatrix} E_{BUS\text{-}m} \\ E_{BUS\text{-}p} \end{vmatrix} \qquad (3)$$

Where the self and mutual values of YA and YD are those identified only with the nodes to be retained and removed respectively and YC=YBt is composed of only the corresponding mutual admittance values, that are common to the nodes m and p.
Now, for the p nodes to be eliminated, it is necessary that, each element of the vector IBUS-p should be zero. Thus we have from (3):

$$I_{BUS\text{-}m} = Y_A\, E_{BUS\text{-}m} + Y_B\, E_{BUS\text{-}p}$$
$$I_{BUS\text{-}p} = Y_C\, E_{BUS\text{-}m} + Y_D\, E_{BUS\text{-}p} = 0 \qquad \textbf{(4)}$$

Solving,

$$\mathbf{E_{BUS\text{-}p} = -\, YD^{-1} Y_C\, E_{BUS\text{-}m}} \qquad \textbf{(5)}$$

Thus, by simplification, we obtain an expression similar to (2) as,

$$\mathbf{I_{BUS\text{-}m}} = \{Y_A - Y_B Y_D\text{-}1 Y_C\}\, \mathbf{E_{BUS\text{-}m}} \qquad \textbf{(6)}$$

Thus by comparing (2) and (6), we get an expression for the new bus admittance matrix in terms of the sub-matrices of the original bus admittance matrix as:

$$\mathbf{Y_{BUSnew} = \{Y_A - Y_B Y_D\ \text{-}1 Y_C\}} \qquad \textbf{(7)}$$

This expression enables us to construct the given network with only the necessary nodes retained and all the unwanted nodes/buses eliminated. However, it can be observed from (7) that the expression involves finding the inverse of the sub-matrix YD (of order p). This would be computationally very tedious if p, the nodes to be eliminated is very large, especially for real practical systems. In such cases, it is more advantageous to eliminate the unwanted nodes from the given network by considering one node only at a time for elimination, as discussed next.

CASE-B: Separate Elimination of Nodes:

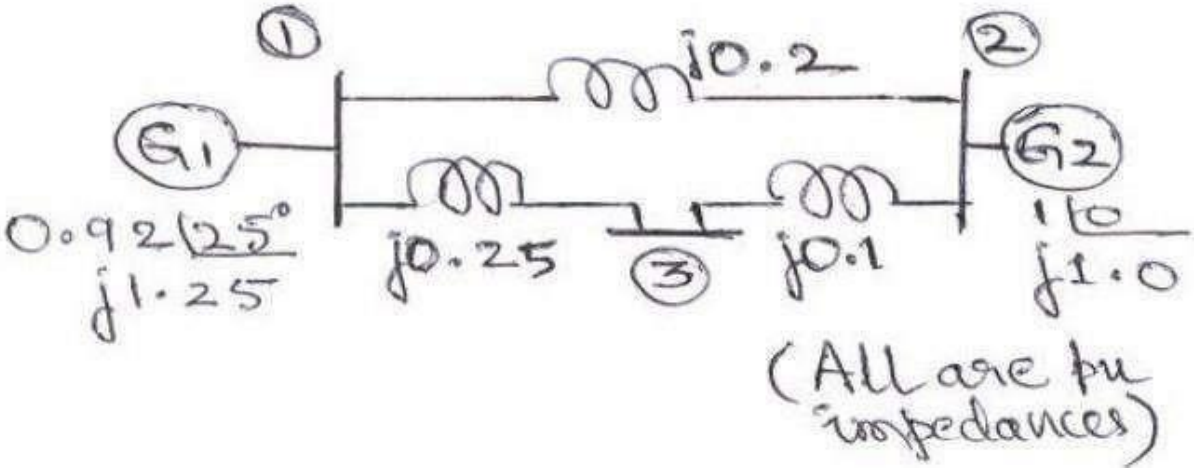
Here again, the system buses are to be renumbered, if necessary, such that the node to be removed always happens to be the last numbered one. The sub-matrix YD then would be a single element matrix and hence it inverse would be just equal to its own reciprocal value. Thus the generalized algorithmic equation for finding the elements of the new bus admittance matrix can be obtained from (6) as,

$\mathbf{Y_{ij}^{\,new} = Y_{ij}^{\,old} - Y_{in} Y_{nj} / Y_{nn}}$ " i,j =1,2,…… n. **(8)**

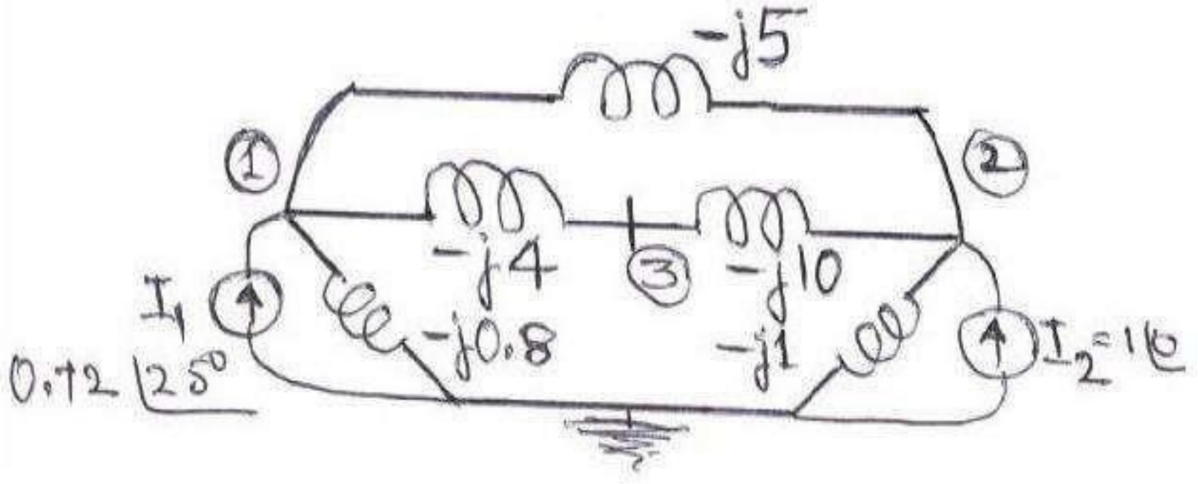
Each element of the original matrix must therefore be modified as per (7). Further, this procedure of eliminating the last numbered node from the given system of n nodes is to be iteratively repeated p times, so as to eliminate all the unnecessary p nodes from the original system.

Examples on Node elimination:

Example-1: Obtain YBUS for the impedance network shown below by the rule of inspection. Also, determine YBUS for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.



The admittance equivalent network is as follows:



The bus admittance matrix is obtained by RoI as:

$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$

The reduced matrix after elimination of node 3 from the given system is determined as per the equation:

$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$$Y_{BUS}^{new} = \begin{array}{c|cc} n/n & 1 & 2 \\ \hline 1 & -j8.66 & j7.86 \\ 2 & j7.86 & -j8.66 \end{array}$$

Alternatively,

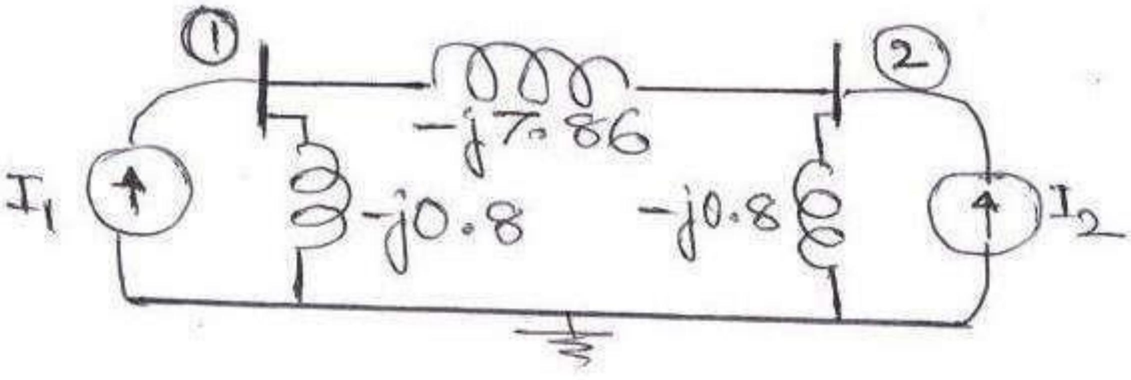
$$Y_{ij}^{new} = Y_{ij}^{old} - Y_{i3} Y_{3j} / Y_{33} \qquad \forall \text{ i,j} = 1,2.$$

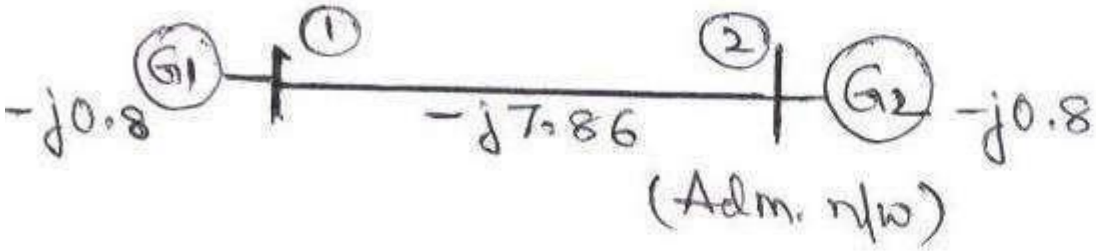
$$Y_{11} = Y_{11} - Y_{13}Y_{31} / Y_{33} = -j8.66$$

$$Y_{22} = Y_{22} - Y_{23}Y_{32} / Y_{33} = -j8.66$$

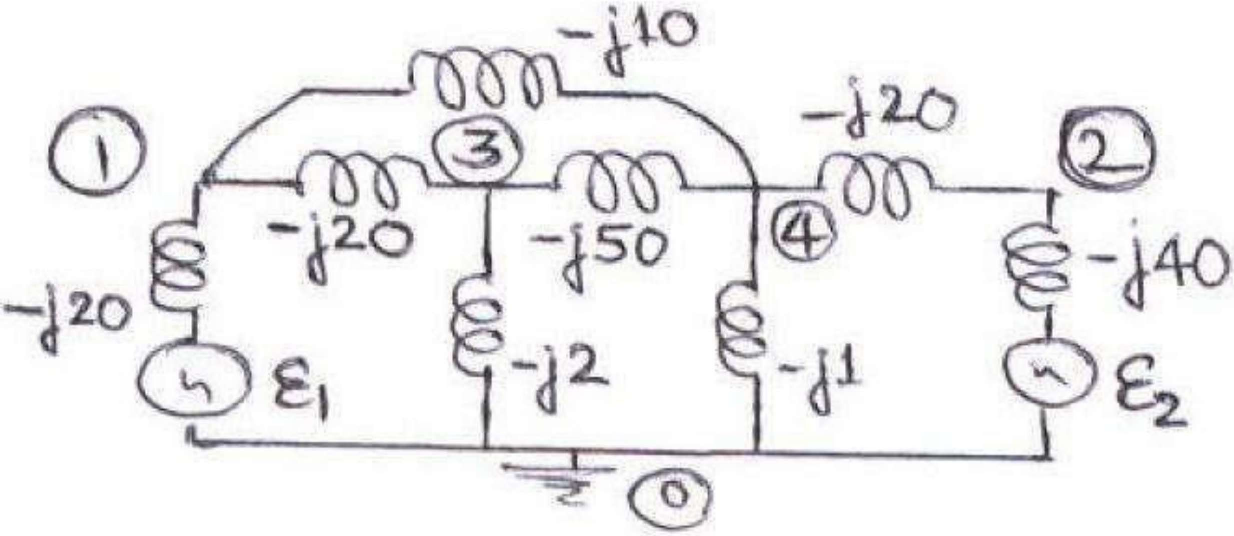
$$Y_{12} = Y_{21} = Y_{12} - Y_{13}Y_{32} / Y_{33} = j7.86$$

Thus the reduced network can be obtained again by the rule of inspection as shown below.





Example-2: Obtain YBUS for the admittance network shown below by the rule of inspection. Also, determine YBUS for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.

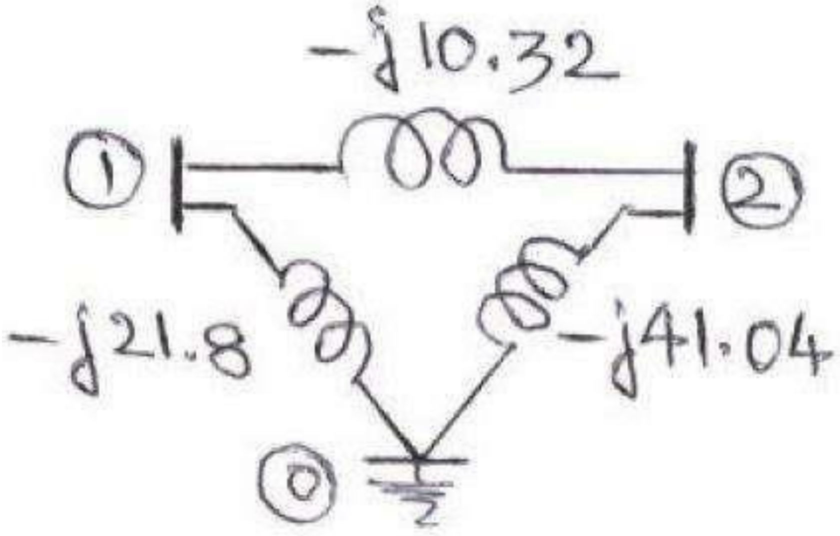


$$Y_{BUS} = \begin{array}{c|c|c|c|c|} n/n & 1 & 2 & 3 & 4 \\ \hline 1 & -j50 & 0 & j20 & j10 \\ \hline 2 & 0 & -j60 & 0 & j72 \\ \hline 3 & j20 & 0 & -j72 & j50 \\ \hline 4 & j10 & j72 & j50 & -j81 \\ \hline \end{array} = \begin{vmatrix} Y_A & Y_B \\ Y_C & Y_D \end{vmatrix}$$

$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$$Y_{BUS}^{new} = \begin{array}{c|c|c|} n/n & 1 & 2 \\ \hline 1 & -j32.12 & j10.32 \\ \hline 2 & j10.32 & -j51.36 \\ \hline \end{array}$$

Thus the reduced system of two nodes can be drawn by the rule of inspection as under:


-j10.32
①
②
-j21.8
-j41.04
⓪

UNIT II

FORMATION OF Z-BUS

ZBUS building

FORMATION OF BUS IMPEDANCE MATRIX

The bus impedance matrix is the inverse of the bus admittance matrix. An alternative method is possible, based on an algorithm to form the bus impedance matrix directly from system parameters and the coded bus numbers. The bus impedance matrix is formedadding one element at a time to a partial network of the given system. The performance equation of the network in bus frame of reference in impedance form using the currents as independent variables is given in matrix form by

$$\overline{E}_{bus} = [Z_{bus}]\overline{I}_{bus} \qquad (9)$$

When expanded so as to refer to a *n* bus system, (9) will be of the form

$$E_1 = Z_{11}I_1 + Z_{12}I_2 + \ldots\ldots + .Z_{1k}I_k \ldots + Z_{1n}I_n$$

$$\vdots$$

$$\vdots$$

$$E_k = Z_{k1}I_1 + Z_{k2}I_2 + \ldots\ldots + Z_{kk}I_k + \ldots. + Z_{kn}I_n$$

$$\vdots$$

$$\vdots$$

$$E_n = Z_{n1}I_1 + Z_{n2}I_2 + \ldots\ldots\ldots + Z_{nk}I_k + \ldots.. + Z_{nn}I_n \qquad (10)$$

Now assume that the bus impedance matrix Zbus is known for a partial network of *m* buses and a known reference bus. Thus, Zbus of the partial network is of dimension *m*x*m*. If now a new element is added between buses *p* and *q* we have the following twopossibilities:

(i) *p* is an existing bus in the partial network and *q* is a new bus; in this case *p*-*q* is a **branch** added to the p-network as shown in Fig 1a, and

(ii) both *p* and *q* are buses existing in the partial network; in this case *p*-*q* is a **link** added to the p-network as shown in Fig 1b.

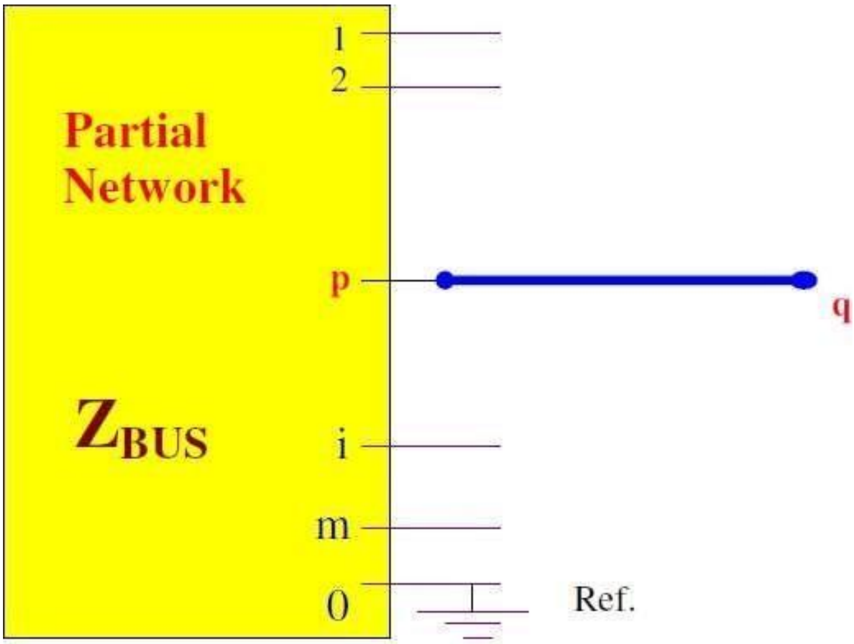


Fig 1a. Addition of branch p-q

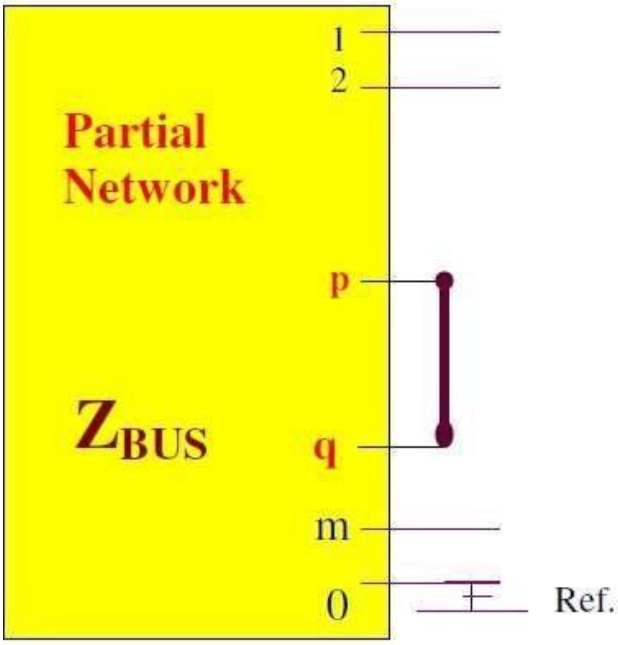


Fig 1b. Addition of link p-q

If the added element ia a branch, p-q, then the new bus impedance matrix would be of order m+1, and the analysis is confined to finding only the elements of the new row and column (corresponding to bus-q) introduced into the original matrix. If the added element ia a link, p-q, then the new bus impedance matrix will remain unaltered with regard to its order. However, all the elements of the original matrix are updated to take account of the effect of the link added.

ADDITION OF A BRANCH

Consider now the performance equation of the network in impedance form with the added branch p-q, given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix} \qquad (11)$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

Vector ypq-rs is not equal to zero and **Zij= Zji** " i,j=1,2,…m,q
(12)

To find Zqi:

The elements of last row-q and last column-q are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.2. Since all other bus currents are zero, we have from (11) that

Ek = Zki Ii = Zki " k = 1, 2,…i .... p,….m, q
(13)
Hence, Eq = Zqi ; Ep = Zpi ………

Also, Eq=Ep -vpq ; so that Zqi = Zpi - vpq " i =1, 2,…i…....p,….m, _q
(14)

To find vpq:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pq} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & \bar{y}_{pq,rs} \\ \bar{y}_{rs,pq} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pq} \\ \bar{v}_{rs} \end{bmatrix} \qquad (15)$$

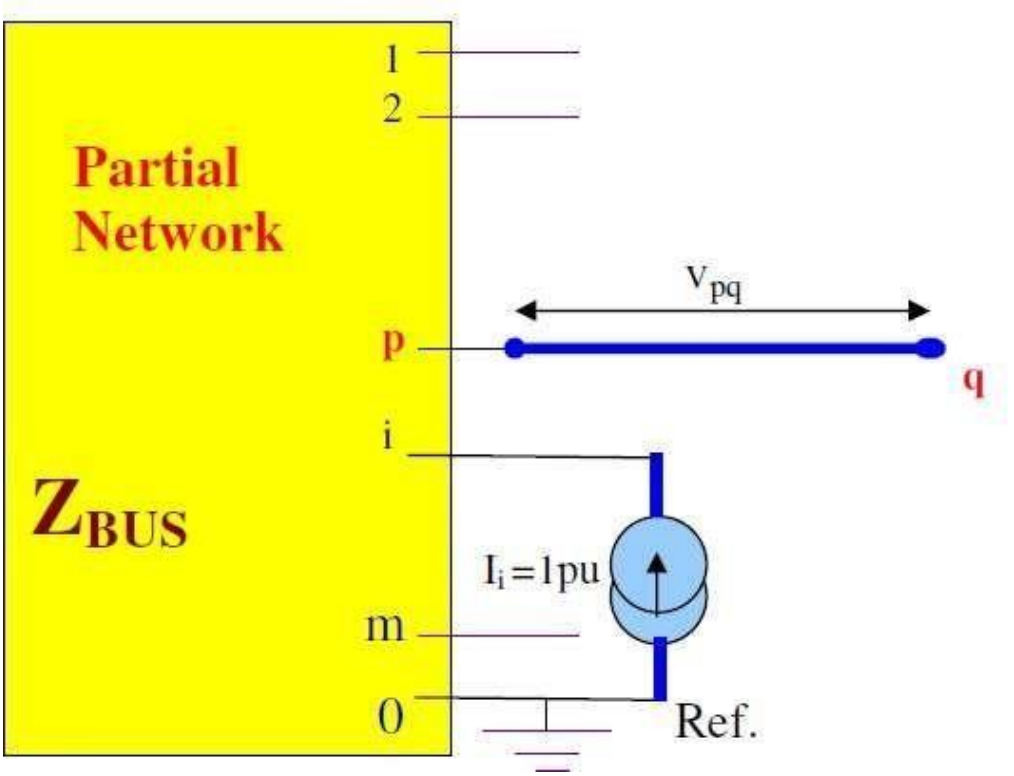


Fig.2 Calculation for Zqi

where i_{pq} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pq} is voltage across element *p-q*

$y_{pq,pq}$ is self – admittance of the added element

$\bar{y}_{pq,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pq}$ is transpose of $\bar{y}_{pq,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-q, is zero, $i_{pq} = 0$. We thus have from (15),

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,rs} \bar{v}_{rs} = 0 \tag{16}$$

Solving, $v_{pq} = -\frac{\overline{y}_{pq,rs}\overline{v}_{rs}}{y_{pq,pq}}$ or

$$v_{pq} = -\frac{\overline{y}_{pq,rs}\left(\overline{E}_r - \overline{E}_s\right)}{y_{pq,pq}} \tag{17}$$

Using (13) and (17) in (14), we get

$$Z_{qi} = Z_{pi} + \frac{\overline{y}_{pq,rs}\left(\overline{Z}_{ri} - \overline{Z}_{si}\right)}{y_{pq,pq}} \quad i = 1,2.....m; i \neq q \tag{18}$$

To find zqq:

The element Z_{qq} can be computed by injecting a current of 1pu at bus-q, $I_q = 1.0$ pu. As before, we have the relations as under:

$E_k = Z_{kq} I_q = Z_{kq}$ ∀ k = 1, 2,…i…....p,….m, q (19)

Hence, $E_q = Z_{qq}$; $E_p = Z_{pq}$; Also, $E_q = E_p - v_{pq}$; so that $Z_{qq} = Z_{pq} - v_{pq}$ (20)

Since now the current in the added element is $i_{pq} = -I_q = -1.0$, we have from (15)

$$i_{pq} = y_{pq,pq}v_{pq} + \overline{y}_{pq,rs}\overline{v}_{rs} = -1$$

Solving, $v_{pq} = -1 + \frac{\overline{y}_{pq,rs}\overline{v}_{rs}}{y_{pq,pq}}$

$$v_{pq} = -1 + \frac{\overline{y}_{pq,rs}\left(\overline{E}_r - \overline{E}_s\right)}{y_{pq,pq}} \tag{21}$$

Using (19) and (21) in (20), we get

$$Z_{qq} = Z_{pq} + \frac{1 + \overline{y}_{pq,rs}\left(\overline{Z}_{rq} - \overline{Z}_{sq}\right)}{y_{pq,pq}} \tag{22}$$

Special Cases

The following special cases of analysis concerning ZBUS building can be considered with respect to the addition of branch to a p-network.

Case (a): If there is no mutual coupling then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then E_p=0. thus,

$$Z_{pi} = 0 \qquad i = 1,2......m : i \neq q$$

And $$Z_{pq} = 0.$$

Hence, from (18) (22) $$Z_{qi} = 0 \qquad i = 1,2......m ; i \neq q$$

And $$Z_{qq} = z_{pq,pq} \qquad \backslash \quad (23)$$

Case (b): If there is no mutual coupling and if p is not the ref. bus, then, from (18) and (22), we again have,

$$Z_{qi} = Z_{pi}, \quad i = 1,2....m; i \neq q$$

$$Z_{qq} = Z_{pq} + z_{pq,pq} \tag{24}$$

ADDITION OF A LINK

Consider now the performance equation of the network in impedance form with the added link p-l, (p-l being a fictitious branch and l being a fictitious node) given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_l \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{l1} & Z_{l2} & \cdots & Z_{li} & \cdots & Z_{lm} & Z_{ll} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_l \end{bmatrix} \tag{25}$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

Vector y_{pq-rs} is not equal to zero and $Z_{ij}= Z_{ji} \qquad \forall$ i,j=1,2,…m,l. (26)

To find Zli:

The elements of last row-l and last column-l are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.3. Further, the current in the added element is made zero by connecting a voltage source, el in series with element p-q, as shown. Since all other bus currents are zero, we have from (25) that

$$E_k = Z_{ki}\, I_i = Z_{ki} \qquad \forall\ k = 1, 2, \ldots i \ldots\ldots p, \ldots. m, l \tag{27}$$

Hence, $e_l = E_l = Z_{li}$; $E_p = Z_{pi}$; $E_p = Z_{pi}$ ………

Also, $e_l = E_p - E_q - v_{pq}$;

So that $Z_{li} = Z_{pi} - Z_{qi} - v_{pq}$ $\forall$ i=1,2,…i….p,...q,….m, $\neq$l (28)

To find vpq:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pl} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pl,pl} & \bar{y}_{pl,rs} \\ \bar{y}_{rs,pl} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pl} \\ \bar{v}_{rs} \end{bmatrix} \tag{29}$$

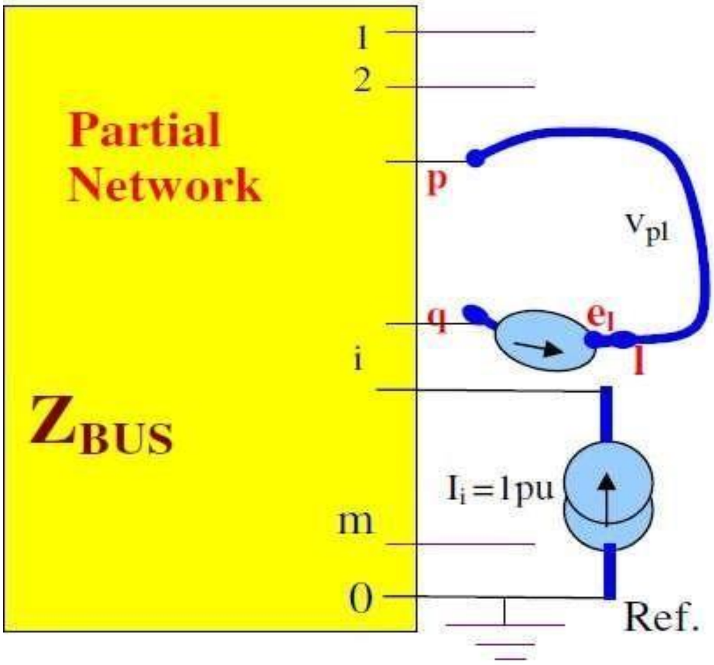


Fig.3 Calculation for Z_{li}

where i_{pl} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pl} is voltage across element *p-q*

$y_{pl,pl}$ is self – admittance of the added element

$\bar{y}_{pl,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pl}$ is transpose of $\bar{y}_{pl,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-l, is zero, $i_{pl} = 0$. We thus have from (29),

$$i_{pl} = y_{pl,pl}v_{pl} + \bar{y}_{pl,rs}\bar{v}_{rs} = 0 \qquad (30)$$

Solving, $v_{pl} = -\dfrac{\bar{y}_{pl,rs}\bar{v}_{rs}}{y_{pl,pl}}$ or

$$v_{pl} = -\frac{\bar{y}_{pl,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pl,pl}} \qquad (31)$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

And $$y_{pl,pl} = y_{pq,pq} \qquad (32)$$

Using (27), (31) and (32) in (28), we get

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{\bar{y}_{pq,rs}\left(\bar{Z}_{ri} - \bar{Z}_{si}\right)}{y_{pq,pq}} \qquad i = 1,2.....m; i \neq l \qquad (33)$$

To find Z_{ll}:

The element Z_{ll} can be computed by injecting a current of 1pu at bus-l, $I_l = 1.0$ pu. As before, we have the relations as under:

$E_k = Z_{kl} I_l = Z_{kl} \quad \forall\ k = 1, 2, \ldots i \ldots\ldots p, \ldots q, \ldots. m, l$ (34)

Hence, $e_l = E_l = Z_{ll}$; $E_p = Z_{pl}$;

Also, $e_l = E_p - E_q - v_{pl}$;

So that $Z_{ll} = Z_{pl} - Z_{ql} - v_{pl} \quad \forall\ i=1,2,\ldots i \ldots. p, \ldots q, \ldots. m, \neq l$ (35)

Since now the current in the added element is $i_{pl} = -I_l = -1.0$, we have from (29)

$$i_{pl} = y_{pl,pl} v_{pl} + \bar{y}_{pl,rs} \bar{v}_{rs} = -1$$

Solving, $v_{pl} = -1 + \dfrac{\bar{y}_{pl,rs} \bar{v}_{rs}}{y_{pl,pl}}$

$$v_{pl} = -1 + \frac{\bar{y}_{pl,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pl,pl}} \tag{36}$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

And $\quad y_{pl,pl} = y_{pq,pq}$ (37)

Using (34), (36) and (37) in (35), we get

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + \bar{y}_{pq,rs}\left(\bar{Z}_{rl} - \bar{Z}_{sl}\right)}{y_{pq,pq}} \tag{38}$$

Special Cases Contd….

The following special cases of analysis concerning Z_{BUS} building can be considered with respect to the addition of link to a p-network.

Case (c): If there is no mutual coupling, then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then E_p=0. thus,

$$Z_{li} = -Z_{qi}, \quad i = 1,2....m; i \neq l$$

$$Z_{ll} = -Z_{ql} + z_{pq,pq} \tag{39}$$

From (39), it is thus observed that, when a link is added to a ref. bus, then the situation is similar to adding a branch to a fictitious bus and hence the following steps are followed:

1. The element is added similar to addition of a branch (case-b) to obtain the new matrix of order m+1.
2. The extra fictitious node, l is eliminated using the node elimination algorithm.

Case (d): If there is no mutual coupling, then elements of *pq rs y* , are zero. Further, if *p* is not the reference node, then

$$Z_{li} = Z_{pi} - Z_{qi}$$

$$\begin{aligned} Z_{ll} &= Z_{pl} - Z_{ql} - z_{pq.pq} \\ &= Z_{pp} + Z_{qq} - 2\,Z_{pq} + z_{pq.pq} \end{aligned} \tag{40}$$

MODIFICATION OF ZBUS FOR NETWORK CHANGES

An element which is not coupled to any other element can be removed easily. The Zbus is modified as explained in sections above, by adding in parallel with the element (to be removed), a link whose impedance is equal to the negative of the impedance of the element to be removed. Similarly, the impedance value of an element which is not coupled to any other element can be changed easily. The Zbus is modified again as explained in sections above, by adding in parallel with the element (whose impedance is to be changed), a link element of impedance value chosen such that the parallel equivalent impedance is equal to the desired value of impedance. When mutually coupled elements are removed, the Zbus is modified by introducing appropriate changes in the bus currents of the original network to reflect the changes introduced due to the removal of the elements.

Examples on ZBUS building

Example 1: For the positive sequence network data shown in table below, obtain ZBUS by building procedure.

Sl. No.	p-q (nodes)	Pos. seq. reactance in pu
1	0-1	0.25
2	0-3	0.20
3	1-2	0.08
4	2-3	0.06

Solution:

The given network is as shown below with the data marked on it. Assume the elements to be added as per the given sequence: 0-1, 0-3, 1-2, and 2-3.

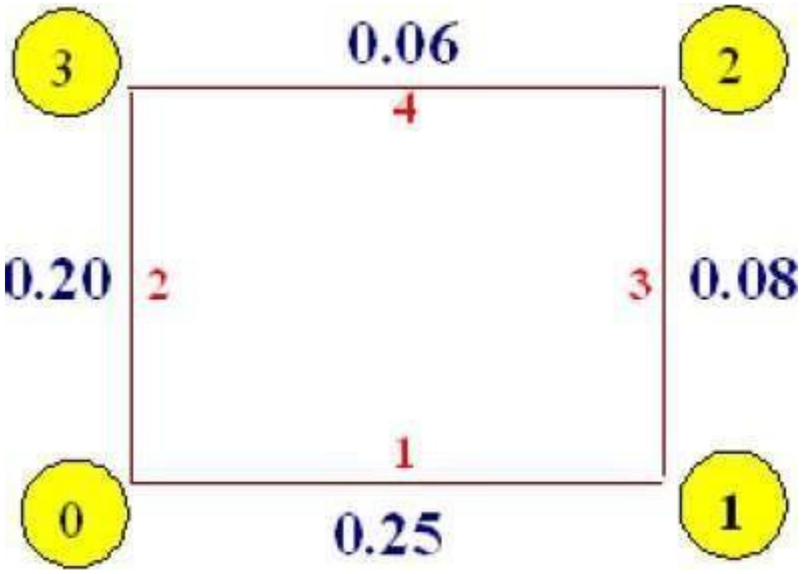
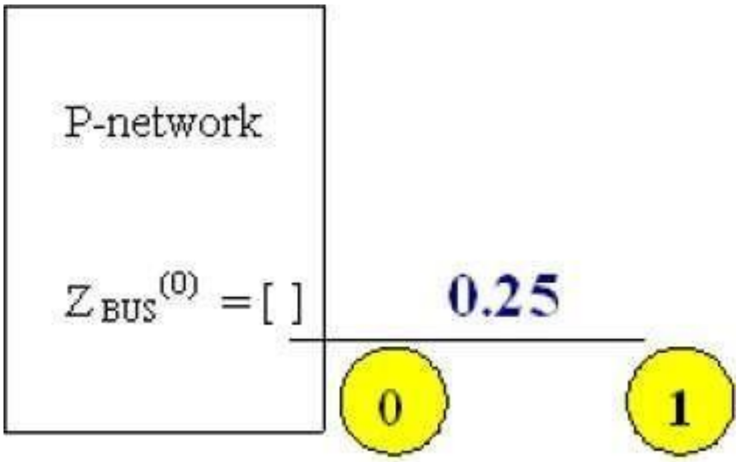


Fig. E1: Example System

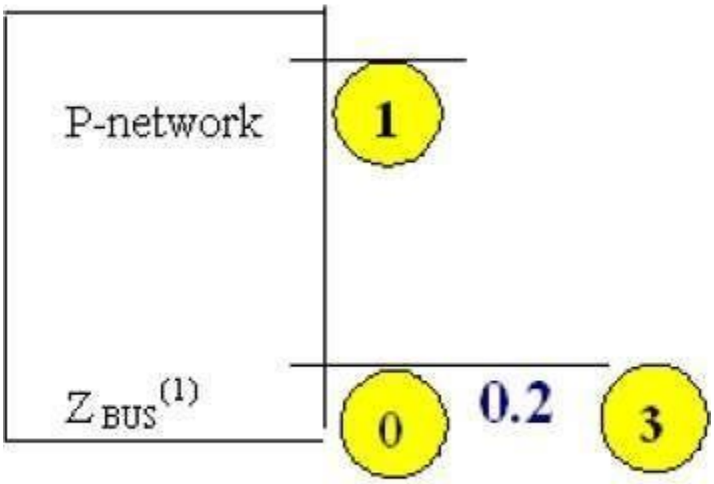
Consider building ZBUS as per the various stages of building through the consideration of the corresponding partial networks as under:

Step-1: Add element–1 of impedance 0.25 pu from the external node-1 (q=1) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



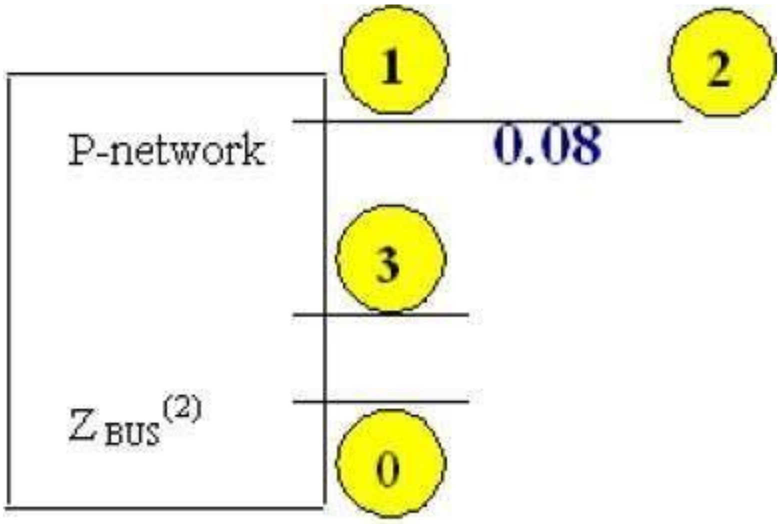
$$Z_{BUS}^{(1)} = \begin{array}{cc} & 1 \\ 1 & \boxed{0.25} \end{array}$$

Step-2: Add element–2 of impedance 0.2 pu from the external node-3 (q=3) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



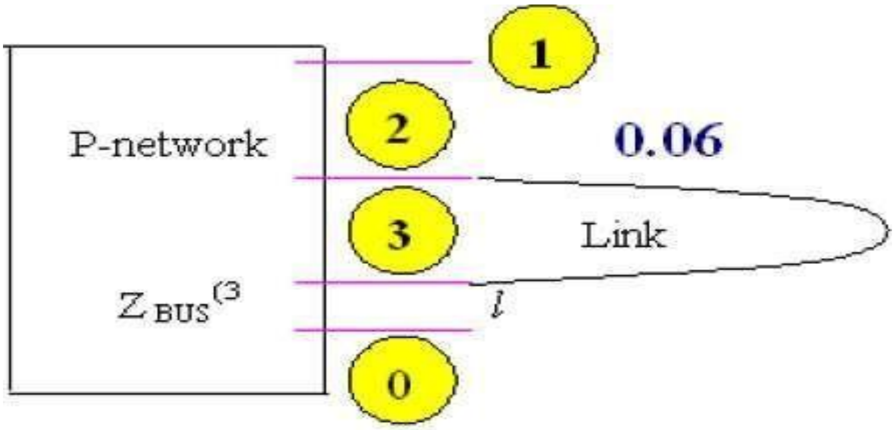
$$Z_{BUS}^{(2)} = \begin{array}{c|c|c|} & 1 & 3 \\ \hline 1 & 0.25 & 0 \\ \hline 3 & 0 & 0.2 \\ \hline \end{array}$$

Step-3: Add element–3 of impedance 0.08 pu from the external node-2 (q=2) to internal node-1 (p=1). (Case-b), as shown in the partial network;



$$Z_{BUS}^{(3)} = \begin{array}{c|c|c|c|} & 1 & 3 & 2 \\ \hline 1 & 0.25 & 0 & 0.25 \\ \hline 3 & 0 & 0.2 & 0 \\ \hline 2 & 0.25 & 0 & 0.33 \\ \hline \end{array}$$

Step-4: Add element–4 of impedance 0.06 pu between the two internal nodes, node-2 (p=2) to node-3 (q=3). (Case-d), as shown in the partial network;

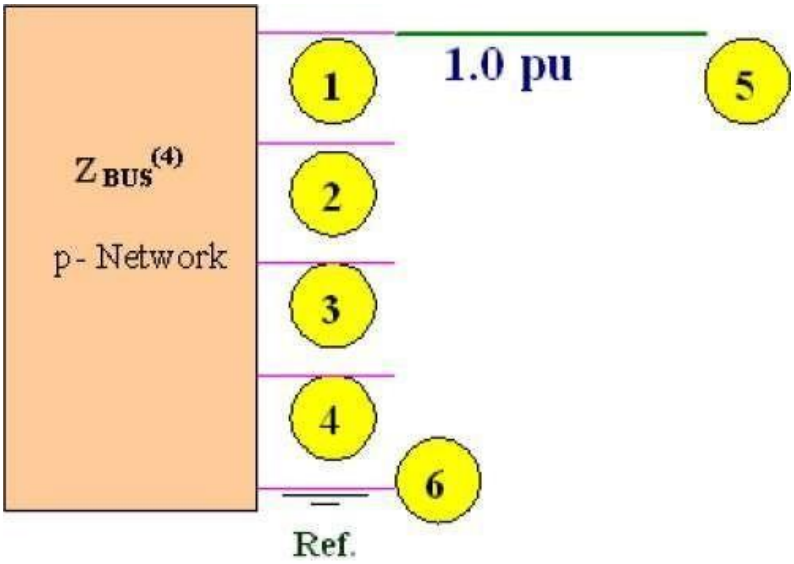


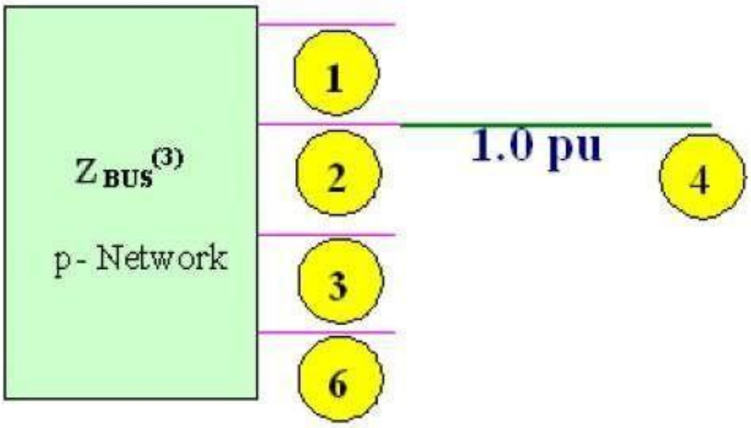
$$Z_{BUS}^{(4)} = \begin{array}{c|cccc} & 1 & 3 & 2 & l \\ \hline 1 & 0.25 & 0 & 0.25 & 0.25 \\ 3 & 0 & 0.2 & 0 & -0.2 \\ 2 & 0.25 & 0 & 0.33 & 0.33 \\ l & 0.25 & -0.2 & 0.33 & 0.59 \end{array}$$

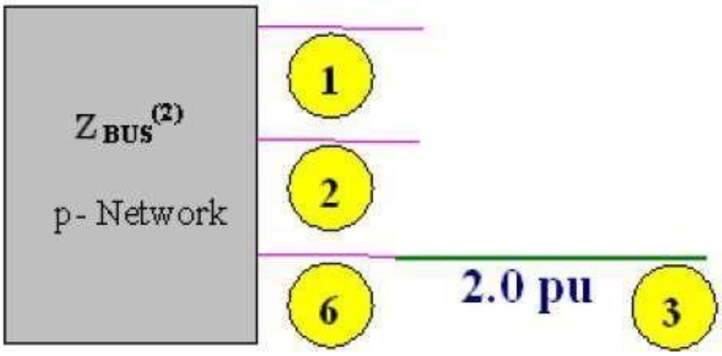
The fictitious node *l* is eliminated further to arrive at the final impedance matrix as under:

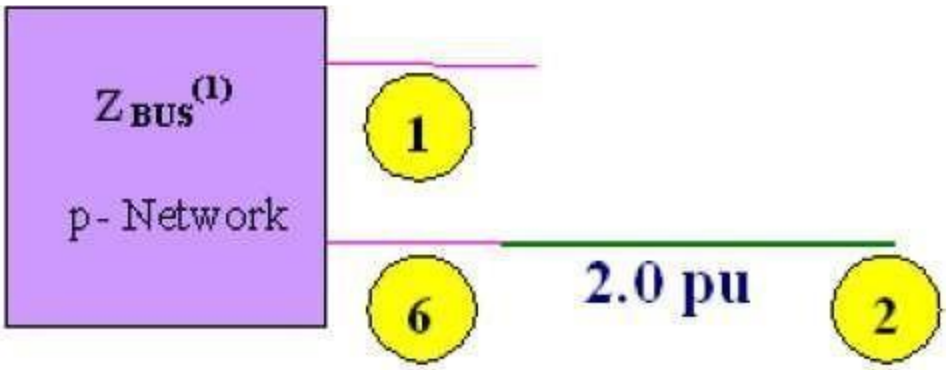
$$Z_{BUS}^{(final)} = \begin{array}{c|ccc} & 1 & 3 & 2 \\ \hline 1 & 0.1441 & 0.0847 & 0.1100 \\ 3 & 0.0847 & 0.1322 & 0.1120 \\ 2 & 0.1100 & 0.1120 & 0.1454 \end{array}$$

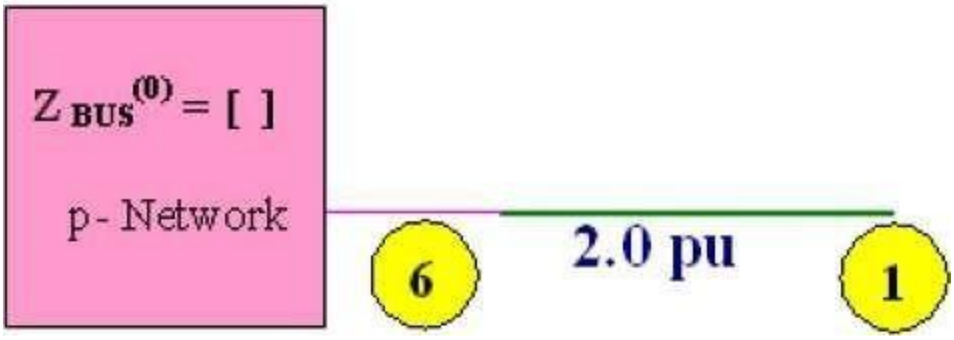
$$Z_{BUS} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 2 & 0 & 0 & 0 & 2 \\ 2 & 0 & 2 & 0 & 2 & 0 \\ 3 & 0 & 0 & 2 & 0 & 0 \\ 4 & 0 & 2 & 0 & 3 & 0 \\ 5 & 2 & 0 & 0 & 0 & 3 \end{array}$$

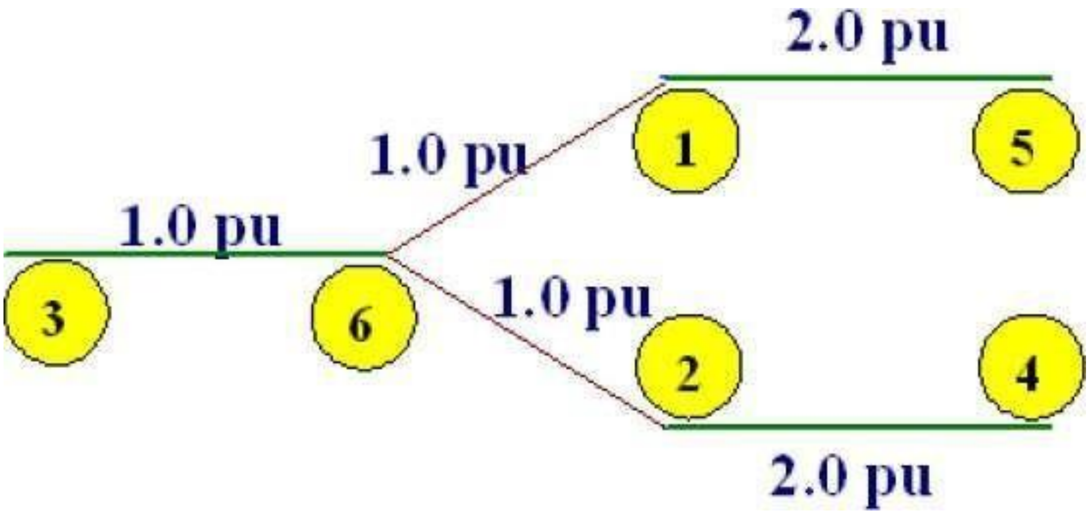

Z BUS(4)
p- Network
1
2
3
4
6
1.0 pu
5
Ref.

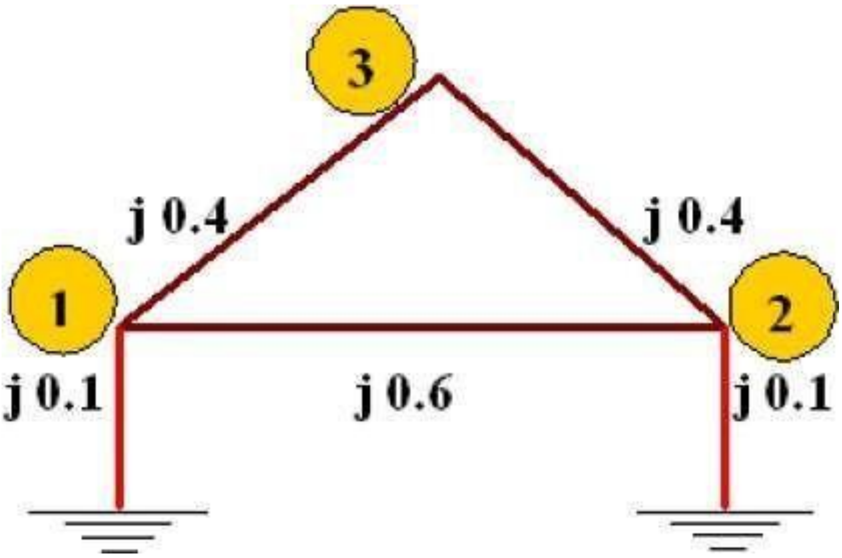

Z BUS(3)
p- Network
1
2
3
6
1.0 pu
4


Z BUS(2)
p- Network
1
2
6
2.0 pu
3


$Z_{BUS}^{(1)}$
p- Network
1
6
2.0 pu
2

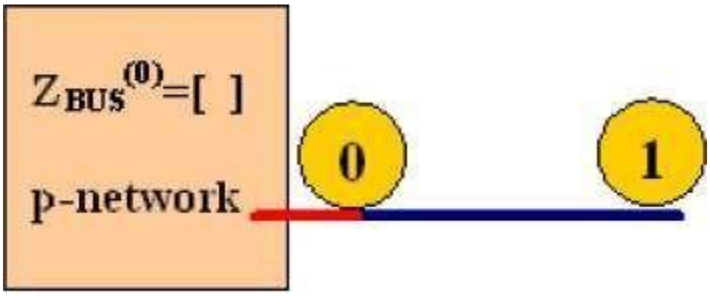

$Z_{BUS}^{(0)} = [\ ]$
p- Network
6
2.0 pu
1


2.0 pu
1.0 pu
1
5
1.0 pu
3
6
1.0 pu
2
4
2.0 pu



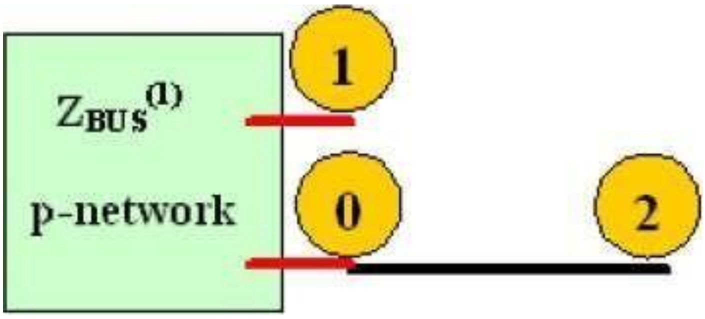
Solution: The specified system is considered with the reference node denoted by node-0. By its inspection, we can obtain the bus impedance matrix by building procedure by following the steps through the p-networks as under:

Step1: Add branch 1 between node 1 and reference node. (q =1, p = 0)



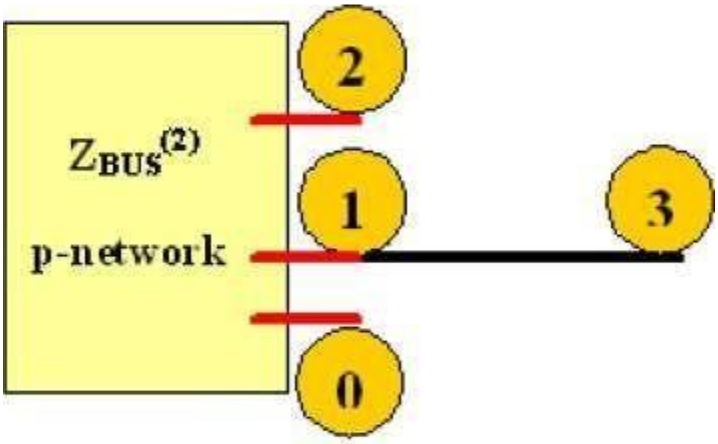
$$Z_{bus}^{(1)} = \begin{array}{cc} & 1 \\ 1 & [j0.1] \end{array}$$

Step2: Add branch 2, between node 2 and reference node. (q = 2, p = 0).



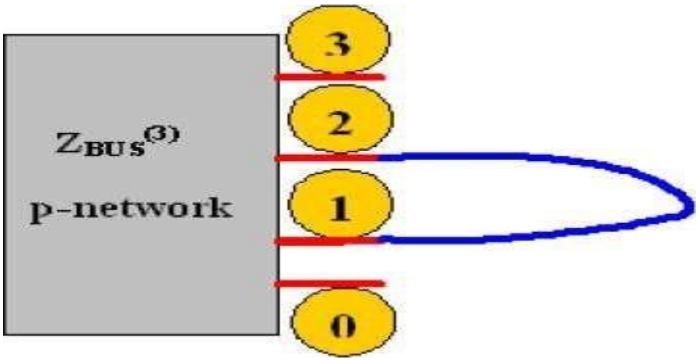
$$Z_{Bus} = \begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} j0.1 & 0 \\ 0 & j0.15 \end{bmatrix} \end{array}$$

Step3: Add branch 3, between node 1 and node 3 (p = 1, q = 3)



$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.1 & 0 & j0.1 \\ 0 & j0.15 & 0 \\ j0.1 & 0 & j0.5 \end{bmatrix} \end{array}$$

Step 4: Add element 4, which is a link between node 1 and node 2. (p = 1, q = 2)



$$Z_{bus} = \begin{array}{c|cccc} & 1 & 2 & 3 & l \\ \hline 1 & j0.1 & 0 & j0.1 & j0.1 \\ 2 & 0 & j0.15 & 0 & -j0.15 \\ 3 & j0.1 & 0 & j0.5 & j0.1 \\ l & j0.1 & -j0.15 & j0.1 & j0.85 \end{array}$$

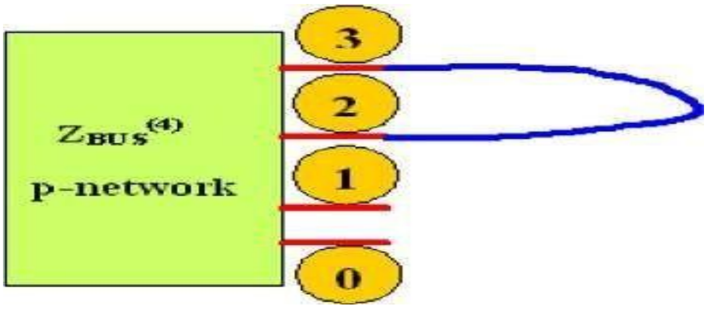
Now the extra node-*l* has to be eliminated to obtain the new matrix of step-4, using the algorithmic relation:

$$\mathbf{Y_{ij}}^{new} = \mathbf{Y_{ij}}^{old} - \mathbf{Y_{in}}\,\mathbf{Y_{nj}} / \mathbf{Y_{nn}} \qquad \forall\ i,j = 1,2,3.$$

$$Z_{bus} = \begin{array}{ccc} 1 & 2 & 3 \\ \end{array}$$

$$Z_{bus} = \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 \\ j0.01765 & j0.12353 & j0.01765 \\ j0.08823 & j0.01765 & j0.48823 \end{bmatrix}$$

Step 5: Add link between node 2 and node 3 (p = 2, q=3)



$$Z_{l1} = Z_{21} - Z_{31} = j0.01765 - j0.08823 = -j0.07058$$
$$Z_{l2} = Z_{22} - Z_{32} = j0.12353 - j0.01765 = j0.10588$$
$$Z_{l3} = Z_{23} - Z_{33} = j0.01765 - j0.48823 = -j0.47058$$
$$Z_{ll} = Z_{2l} - Z_{3l} + z_{23,23}$$
$$= j0.10588 - (-j0.47058) + j0.4 = j0.97646$$

Thus, the new matrix is as under:

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 1 \end{array} \\ \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 & -j0.07058 \\ j0.01765 & j0.12353 & j0.01765 & j0.10588 \\ j0.08823 & j0.01765 & j0.48823 & -j0.47058 \\ -j0.07058 & j0.10588 & -j0.47058 & j0.97646 \end{bmatrix} \end{array}$$

Node l is eliminated as shown in the previous step:

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.08313 & j0.02530 & j0.05421 \\ j0.02530 & j0.11205 & j0.06868 \\ j0.05421 & j0.06868 & j0.26145 \end{bmatrix} \end{array}$$

Further, the bus admittance matrix can be obtained by inverting the bus impedance matrix as under:

$$Y_{bus} = [Z_{bus}]^{-1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -j14.1667 & j1.6667 & j2.5 \\ j1.6667 & -j10.8334 & j2.5 \\ j2.5 & j2.5 & -j5.0 \end{bmatrix} \end{array}$$

As a check, it can be observed that the bus admittance matrix, Y_{BUS} can also be obtained by the rule of inspection to arrive at the same answer.

UNIT III

FAULT ANALYSIS

INTRODUCTION

A fault is any abnormal condition in a power system. The steady state operating mode of a power system is balanced 3-phase a.c. .However, due to sudden external or internal changes in the system, this condition is disrupted.

When the insulation of the system fails at one or more points or a conducting object comes into contact with a live point, a short circuit or a fault occurs.

CAUSES OF POWER SYSTEM FAULTS

The causes of faults are numerous, e.g.

- Lightning
- Heavy winds
- Trees falling across lines
- Vehicles colliding with towers or poles
- Birds shorting lines
- Aircraft colliding with lines
- Vandalism
- Small animals entering switchgear
- Line breaks due to excessive loading

COMMON POWER SYSTEM FAULTS

Power system faults may be categorised as one of four types; in order of frequency of occurrence, they are:

- Single line to ground fault
- Line to line fault
- Double line to ground fault
- Balanced three phase fault

The first three types constitutes severe unbalanced operating conditions which involves only one or two phases hence referred to as unsymmetrical faults. In the fourth type, a faultinvolving all the three phases occurs therefore referred to as symmetrical (balanced) fault.

1.04 EFFECTS OF POWER SYSTEM FAULTS

Faults may lead to fire breakout that consequently results into loss of property, loss of life and destruction of a power system network. Faults also leads to cut of supply in areas beyond the fault point in a transmission and distribution network leading to power blackouts; this interferes with industrial and commercial activities that supports economic growth, stalls learning activities in institutions, work in offices, domestic applications and creates insecurity at night.

All the above results into retarded development due to low gross domestic product realised.

It is important therefore to determine the values of system voltages and currents during faulted conditions, so that protective devices may be set to detect and minimize the harmful effects of such contingencies

THEVENIN'S EQUIVALENT CIRCUIT

Thevenin's theorem states that any linear network containing any number of voltage sources and impedances can be replaced by a single emf and an impedance.

The emf is the open circuit voltage as seen from the terminals under consideration and the impedance is the network impedance as seen from these terminals.

This circuit consisting of a single emf and impedance is known as Thevenin's equivalent circuit.

The calculation of fault current can then be very easily done by applying this theorem after obtaining the open circuit emf and network impedance as seen from the fault point.

SYMMETRICAL COMPONENTS

The majority of faults in power systems are asymmetrical. To analyse an asymmetrical fault, an unbalanced 3- phase circuit has to be solved. Since the direct solution of such a circuit is very difficult, the solution can be more easily obtained by using symmetrical components since this yields three (fictitious) single phase networks, only one of which contains a driving emf.

Since the system reactances are balanced the thee fictitious networks have no mutual coupling between them, a fact that is making this method of analysis quite simple.

1.21 General principles

Any set of unbalanced 3-phase voltages (or current) can be transformed into 3 balanced sets. These are:

1. A positive sequence set of three symmetrical voltages (i.e. all numerically equal and all displaced from each other by $120^{0)}$ having the same phase sequence *abc* as the original set and denoted by $V_{a1,}V_{b1,}V_{c1}$ as shown in the fig(1a)

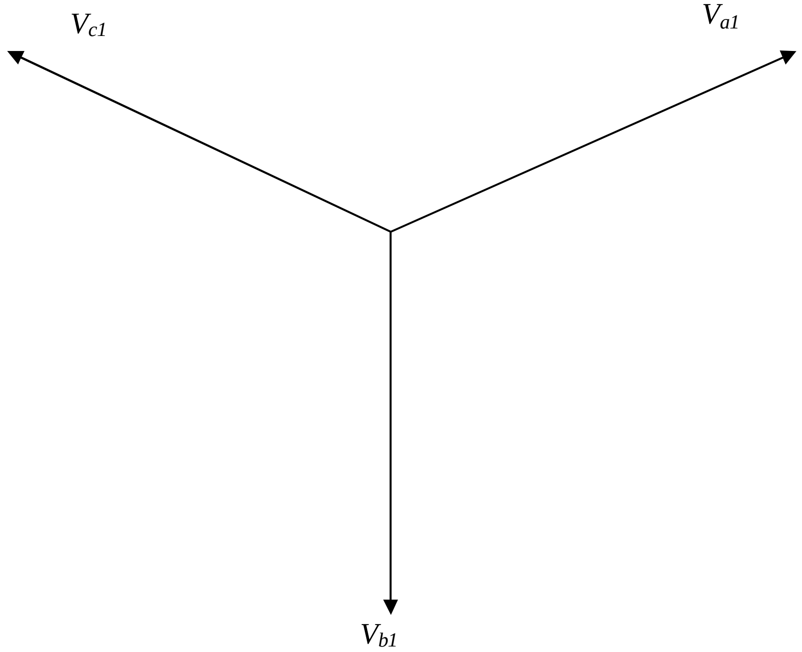


Fig. (a)

2. A negative sequence set of three symmetrical voltages having the phase sequence opposite to that of the original set and denoted by V_{a2}, V_{b2}, V_{c2} as shown in fig(1b)

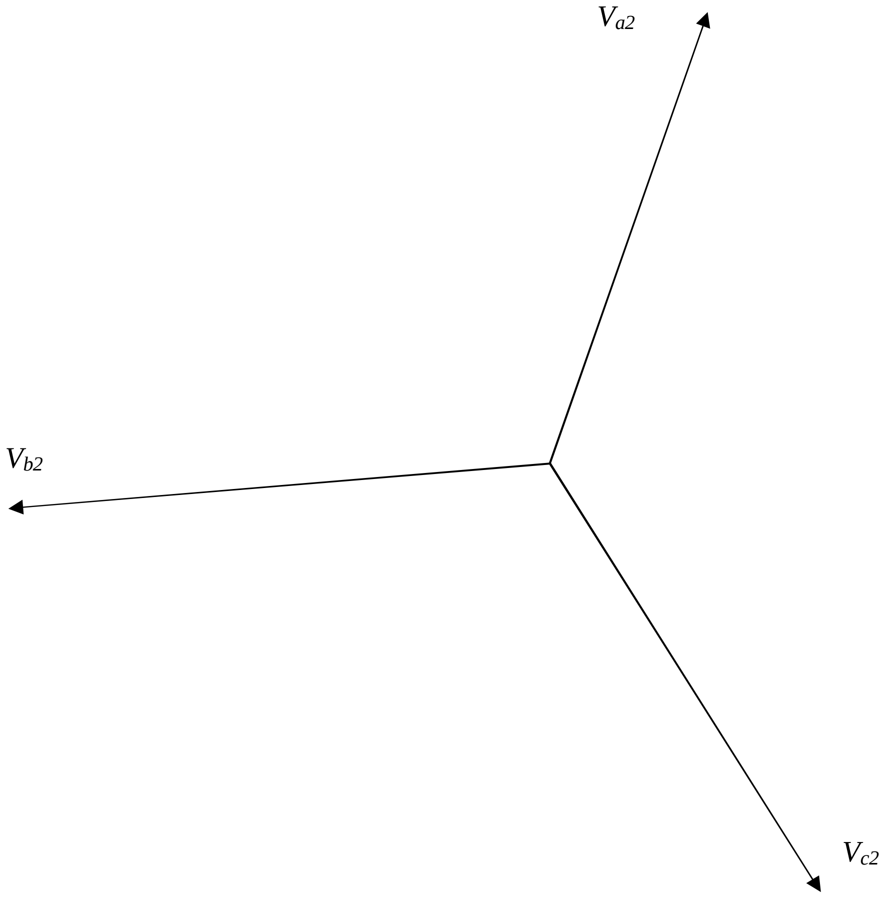


Fig. 1 (b)

3. A zero sequence set of three voltages, all equal in magnitude and in phase with each other and denoted by V_{a0}, V_{b0}, V_{c0} as shown in fig (1c) below:

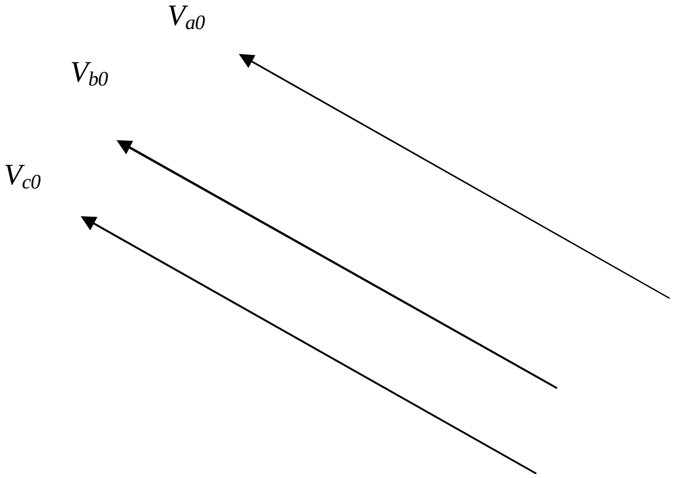


Fig. 1 (c)

The positive, negative and zero sequence sets above are known as symmetrical components.

Thus we have,

$V_a = V_{a1} + V_{a2} + V_{a0}$

$V_b = V_{b1} + V_{b2} + V_{b0}$

$V_c = V_{c1} + V_{c2} + V_{c0}$

The symmetrical components application to power system analysis is of fundamental importance since it can be used to transform arbitrarily unbalanced condition into symmetrical components, compute the system response by straightforward circuit analysis on simple circuit models and transform the results back to the original phase variables.

Generally the subscripts 1, 2 and 0 are used to indicate positive sequence, negative sequence and zero sequence respectively.

The symmetrical components do not have separate existence; they are just mathematical components of unbalanced currents (or voltages) which actually flow in the system.

1.2.2 The "a" operator

The operator "a" as used in symmetrical components is one in which when multiplied to a vector, rotates the vector through 120^0 in a positive (anticlockwise) direction without changing the magnitude.

The operator "a" is defined as $1 \angle 120^0$

THREE-SEQUENCE IMPEDANCES AND SEQUENCE NETWORKS

Positive sequence currents give rise to only positive sequence voltages, the negative sequence currents give rise to only negative sequence voltages and zero sequence currents give rise to only zero sequence voltages, hence each network can be regarded as flowing within in its own network through impedances of its own sequence only.

In any part of the circuit, the voltage drop caused by current of a certain sequence depends on the impedance of that part of the circuit to current of that sequence.

The impedance of any section of a balanced network to current of one sequence may be different from impedance to current of another sequence.

The impedance of a circuit when positive sequence currents are flowing is called impedance,

When only negative sequence currents are flowing the impedance is termed as negative sequence impedance.

With only zero sequence currents flowing the impedance is termed as zero sequence impedance.

The analysis of unsymmetrical faults in power systems is carried out by finding the symmetrical components of the unbalanced currents. Since each sequence current causes a voltage drop of that sequence only, each sequence current can be considered to flow in an independent network composed of impedances to current of that sequence only.

The single phase equivalent circuit composed of the impedances to current of any onesequence only is called the sequence network of that particular sequence.

The sequence networks contain the generated emfs and impedances of like sequence.

Therefore for every power system we can form three- sequence network s. These sequence networks, carrying current I_{a1}, I_{a2} and I_{a0} are then inter-connected to represent the different fault conditions.

PHYSICAL SIGNIFICANCE OF SEQUENCE COMPONENTS

This is achieved by considering the fields which results when these sequence voltages are applied to the stator of a 3-phase machine e.g. an induction motor.

If a positive sequence set of voltages is applied to the terminals a, b, c of the machine, a magnetic field revolving in a certain direction will be set up. If now the voltages to the terminals band c are changed by interchanging the leads to terminals b and c, it is known from induction motor theory that the direction of magnetic field would be reversed.

It is noted that for this condition, the relative phase positions of the voltages applied to the motor are the same as for the negative sequence set.

Hence, a negative sequence set of voltages produces a rotating field rotating in an opposite direction to that of positive sequence.

For both positive and negative sequence components, the standard convention of counter clockwise rotation is followed.

The application of zero sequence voltages does not produce any field because these voltages are in phase and the three -phase windings are displaced by 120^0.The positive and the negative sequence set are the balanced one. Thus, if only positive and negative sequence currents are flowing, the phasor sum of each will be zero and there will be no residual current. However, the zero sequence components of currents in the three phases are in phase and the residual current will be three times the zero sequence current of one phase. In the case of a fault involving ground, the positive and negative sequence currents are in equilibrium while the zero sequence currents flow through the ground and overhead ground wires.

SEQUENCE NETWORKS OF SYNCHRONOUS MACHINES

An unloaded synchronous machine having its neutral earthed through impedance, $z_{n,}$ is shown in fig. 2(a) below.

A fault at its terminals causes currents I_a, I_b and I_c to flow in the lines. If fault involves earth, a current I_n flows into the neutral from the earth. This current flows through the neutral impedance $Z_{n.}$

Thus depending on the type of fault, one or more of the line currents may be zero.

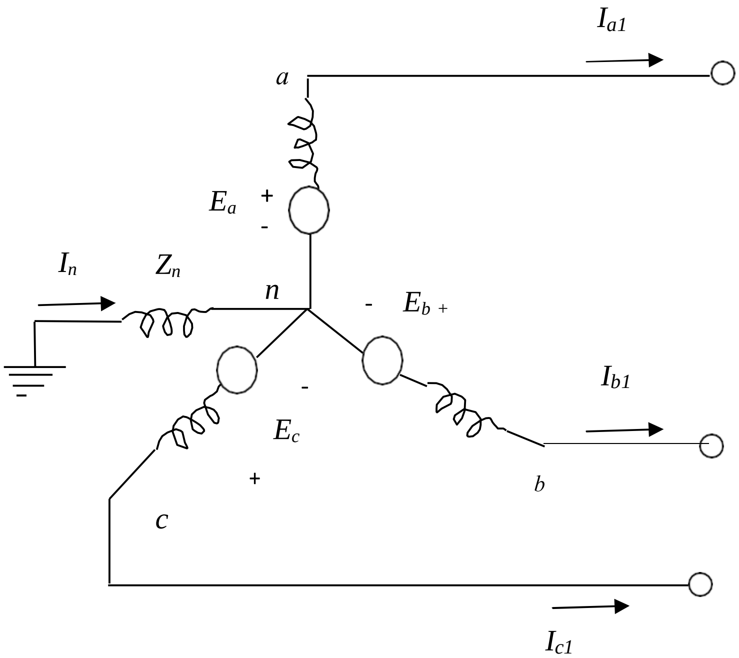


Fig.2 (a)

Positive sequence network

The generated voltages of a synchronous machine are of positive sequence only since the windings of a synchronous machine are symmetrical.

The positive sequence network consists of an emf equal to no load terminal voltages and is in series with the positive sequence impedance Z_1 of the machine. Fig.2 (b) and fig.2(c) shows the paths for positive sequence currents and positive sequence network respectively on a single phase basis in the synchronous machine. The neutral impedance Z_n does not appear in the circuit because the phasor sum of I_{a1}, I_{b1} and I_{c1} is zero and no positive sequence current can flow through Z_n. Since its a balanced circuit, the positive sequence N

The reference bus for the positive sequence network is the neutral of the generator.

The positive sequence impedance Z_1 consists of winding resistance and direct axis reactance. The reactance is the sub-transient reactance X''_d or transient reactance X'_d or synchronous reactance X_d depending on whether sub-transient, transient or steady state conditions are being studied.

From fig.2 (b) , the positive sequence voltage of terminal *a* with respect to the reference bus is given by:

$V_{a1} = E_a - Z_1 I_{a1}$

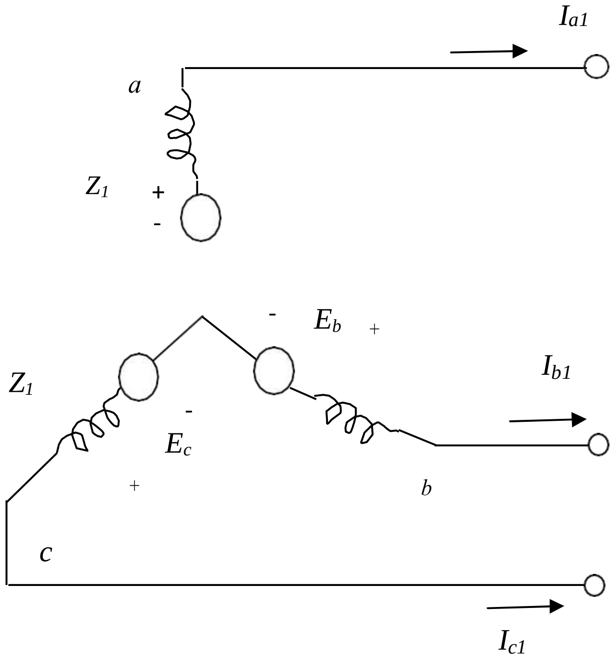


Fig.2 (b)

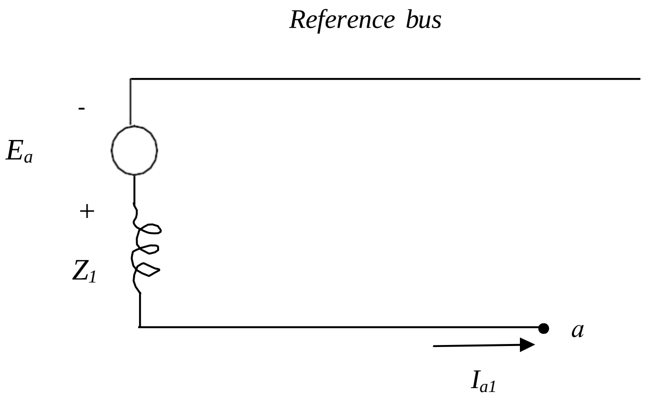


Fig.2(c)

2.02 Negative sequence network

A synchronous machine does not generate any negative sequence voltage. The flow of negative sequence currents in the stator windings creates an mmf which rotates at synchronous speed in a direction opposite to the direction of rotor, i.e., at twice the synchronous speed with respect to rotor.

Thus the negative sequence mmf alternates past the direct and quadrature axis and sets up a varying armature reaction effect. Thus, the negative sequence reactance is taken as the average of direct axis and quadrature axis sub-transient reactance, i.e.,

$X_2 = 0.5\ (X''_d + X''_q)$.

It not necessary to consider any time variation of X_2 during transient conditions because there is no normal constant armature reaction to be effected. For more accurate calculations, the negative sequence resistance should be considered to account for power dissipated in the rotor poles or damper winding by double supply frequency induced currents.

The fig.2 (d) and fig.2 (e) shows the negative sequence currents paths and the negativesequence network respectively on a single phase basis of a synchronous machine.

The reference bus for the negative sequence network is the neutral of the machine. Thus, the negative sequence voltage of terminal *a* with respect to the reference bus is given by:

$V_{a2} = -Z_2 I_{a2}$

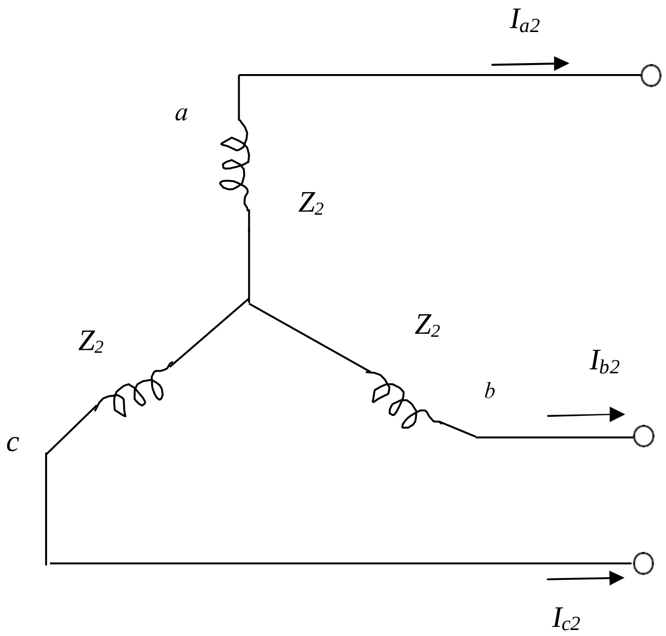


Fig.2 (d)

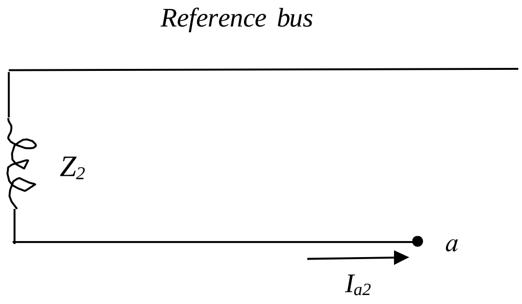


Fig.2 (e)

2.0.3 Zero sequence network

No zero sequence voltage is induced in a synchronous machine. The flow of zero sequence currents in the stator windings produces three mmf which are in time phase. If each phase winding produced a sinusoidal space mmf, then with the rotor removed, the flux at a point on the axis of the stator due to zero sequence current would be zero at every instant.

When the flux in the air gap or the leakage flux around slots or end connections is considered, no point in these regions is equidistant from all the three –phase windings of the stator.

The mmf produced by a phase winding departs from a sine wave, by amounts which depend upon the arrangement of the winding.

The zero sequence currents flow through the neutral impedance Z_n and the current flowing through this impedance is $3I_{a0}$.

Fig.2(f) and fig.2(g) shows the zero sequence current paths and zero sequence network respectively, and as can be seen, the zero sequence voltage drop from point a to ground is $-3I_{a0}Z_n - I_{a0}Zg_0$ where Z_{g0} is the zero sequence impedance per phase of the generator.

Since the current in the zero sequence network is I_{a0} this network must have an impedance of $3Z_n + Z_{g0}$. Thus,

$Z_0 = 3Z_n + Z_{g0}$

The zero sequence voltage of terminal a with respect to the reference bus is thus:

$V_{a0} = -I_{a0}Z_0$

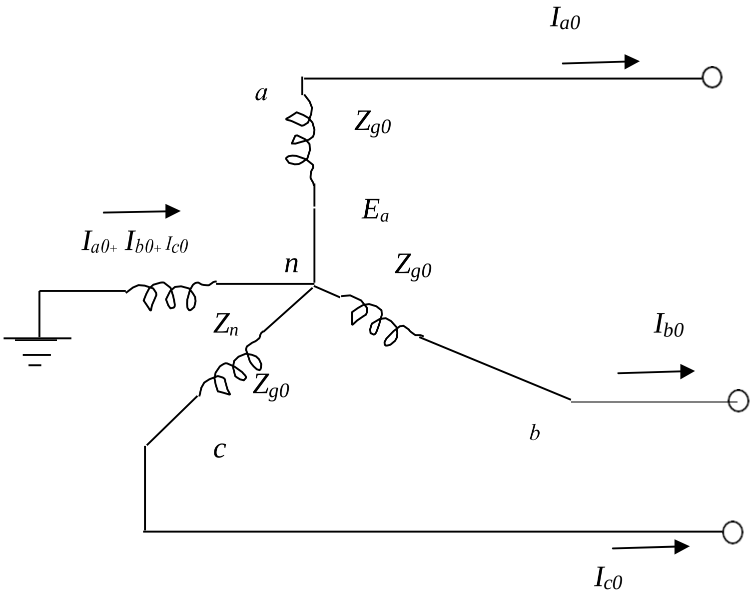

Ia0
a
Zg0
Ea
Ia0+ Ib0+ Ic0
n
Zg0
Zn
Zg0
Ib0
b
c
Ic0

Fig.2 (f)

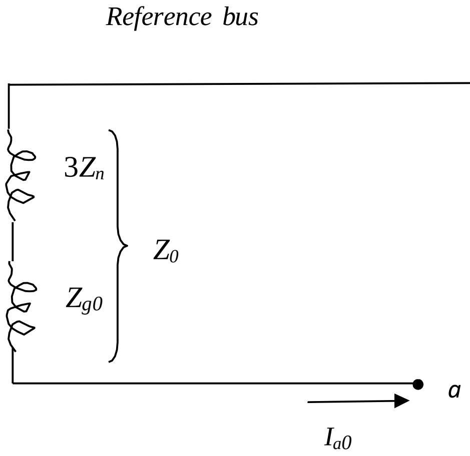

Reference bus
3Zn
Z0
Zg0
a
Ia0

Fig.2 (g)

SEQUENCE IMPEDANCES OF TRANSMISSION LINE

The positive and negative sequence impedances of linear symmetrical static circuits do not depend on the phase sequence and are, therefore equal. When only zero sequence currents flow in the lines, the currents in all the phases are identical. These currents return partly through the ground and partly through overhead ground wires.

The magnetic field due to the flow of zero sequence currents through line, ground and round wires is very different from the magnetic field due to positive sequence currents. The zero sequence reactance of lines is about 2 to 4 times the positive sequence reactance.

SEQUENCE IMPEDANCES OF TRANSFORMERS

A power system network has a number of transformers for stepping up and stepping down the voltage levels.

A transformer for a 3-phase circuit may consist of three single phase transformers with windings suitably connected in star or delta or it may be a 3-phase unit.

Modern transformers are invariably three-phase units because of their lower cost, lesser space requirements and higher efficiency. The positive sequence impedance of a transformer equals its leakage impedance. The resistance of the windings is usually small as compared to leakage reactance.

For transformers above 1 MVA rating, the reactance and impedance are almost equal. Since the transformer is a static device, the negative sequence impedance is equal to the positive sequence impedance.

The zero sequence impedance of 3-phase units is slightly different from positive sequence impedance. However the difference is very slight and the zero sequence impedance is also assumed to be the same as the positive sequence impedance.

The flow of zero sequence currents through a transformer and hence in the system depends greatly on the winding connections. The zero sequence currents can flow through the winding connected in star only if the star point is grounded. If the star point isolated zero sequence currents cannot flow in the winding.

The zero sequence currents cannot flow in the lines connected to a delta connected winding because no return path is available for these zero sequence currents. However, the zero sequence currents caused by the presence of zero sequence voltages can circulate through the delta connected windings.

FORMATION OF SEQUENCE NETWORKS

A power system network consists of synchronous machines, transmission lines and transformers.

The positive sequence network is the same as the single line reactance diagram used for the calculation of symmetrical fault current. The reference bus for positive sequence network is the system neutral.

The negative sequence network is similar to the positive sequence network except that the negative sequence network does not contain any voltage source. The negative sequence impedances for transmission line and transformers are the same as the positive sequence impedances. But the negative sequence impedance of a synchronous machine may be different from its positive sequence impedance.

Any impedance connected between a neutral and ground is not included in the positive and negative sequence networks because the positive and the negative sequence currents cannot flow through such impedance.

The zero sequence network also does not contain any voltage source. Any impedance included between neutral and ground becomes three times its value in a zero sequencenetwork.

The following are the summary of the rules for the formation of sequence networks:-

- The positive sequence network is the same as single line impedance or reactancediagram used in symmetrical fault analysis. The reference bus for this network is the system neutral.

- The generators in power system produce balanced voltages. Therefore only positive sequence network has voltage source. There are no voltage sources in negative and zero sequence networks.
- The positive sequence current can cause only positive sequence voltage drop. Similarly negative sequence current can cause only negative sequence voltage drop and zero sequence current can cause only zero sequence voltage drop.
- The reference for negative sequence network is the system neutral. However, the reference for zero sequence network is the ground. Zero sequence current can flow only if the neutral is grounded.
- The neutral grounding impedance Z_n appears as $3Z_n$ in the zero sequence network.
- The three sequence networks are independent and are interconnected suitably depending on the type of fault.

UNSYMMETRICAL FAULTS

The basic approach to the analysis of unsymmetrical faults is to consider the general situation shown in the fig.3.0 which shows the three lines of the three- phase power system at the point of fault.

The general terminals brought out are for purposes of external connections which simulate the fault. Appropriate connections of the three stubs represent the different faults, e.g., connecting stub *`a'* to ground produces a single line to ground fault, through zero impedance, on phase *`a'*. The currents in stubs *b* and *c* are then zero and *Ia* is the fault current.

Similarly, the connection of stubs b and *c* produces a line to line fault, through zero impedance, between phases *b* and *c*, the current in stub *a* is then zero and I_b is equal to *Ic*.The positive assignment of phase quantities is important. It is seen that the currents flow out of the system.

The three general sequence circuits are shown in fig.3.1 (a). The ports indicated correspond to the general 3- phase entry port of fig.3.1. A suitable inter- connection of the three- sequence networks depending on the type fault yields the solution to the problem.

The sequence networks of fig.3.1 (a) can be replaced by equivalent sequence networks of fig.3.1 (b) . Z_0, Z_1 and Z_2 indicate the sequence impedances of the network looking into the fault

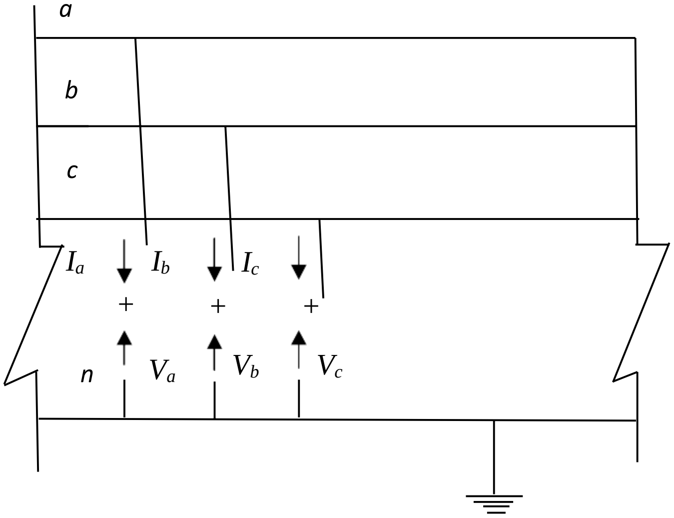


Fig.3.0 General 3- phase access port

General sequence networks

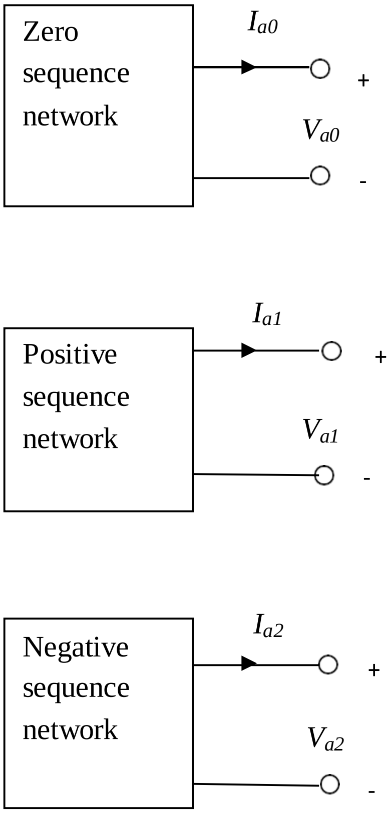


Fig.3.1 (a)

Equivalent sequence networks

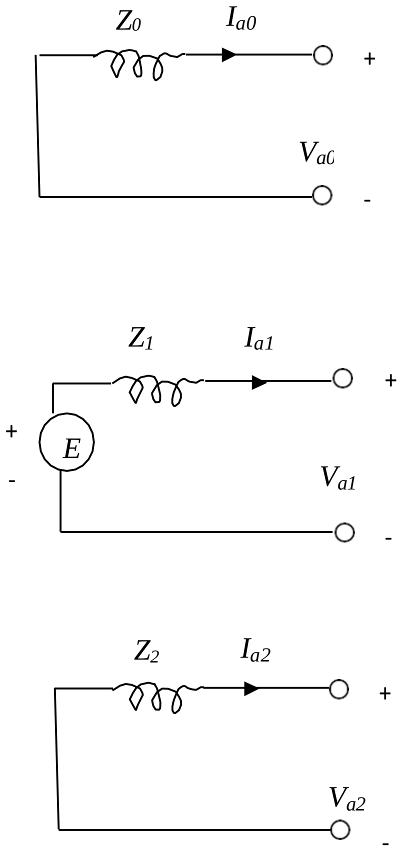


Fig.3.1 (b)

SINGLE LINE TO GROUND FAULT

The termination of the three- phase access port as shown in fig. 3.2 brings about a condition of single line to ground fault through a fault impedance Z_f .

Typically Z_f is set to zero in all fault studies. I include Z_f in the analysis for the sake of generality. The terminal conditions at the fault point give the following equations:

$I_b = 0$

$I_c = 0$

$V_a = I_a Z_f$

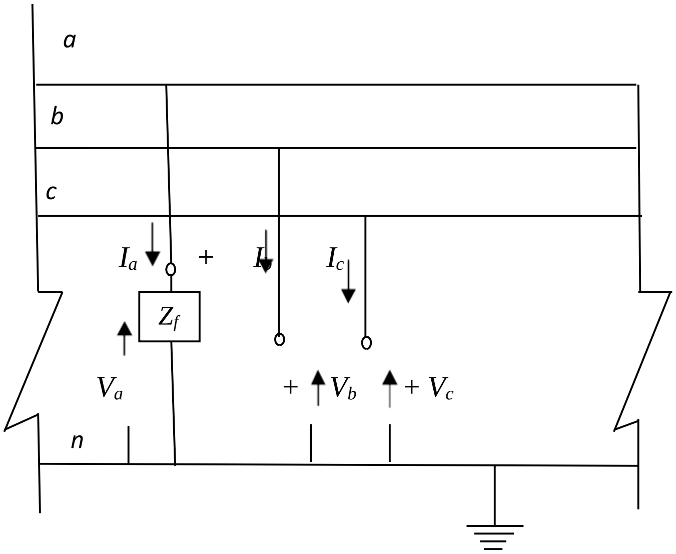


Fig. 3.2

Connections of sequence networks for a single line to ground fault and its simplified equivalent circuit are shown in the fig. 3.3(a) and fig. 3.3 (b) below:

General sequence networks

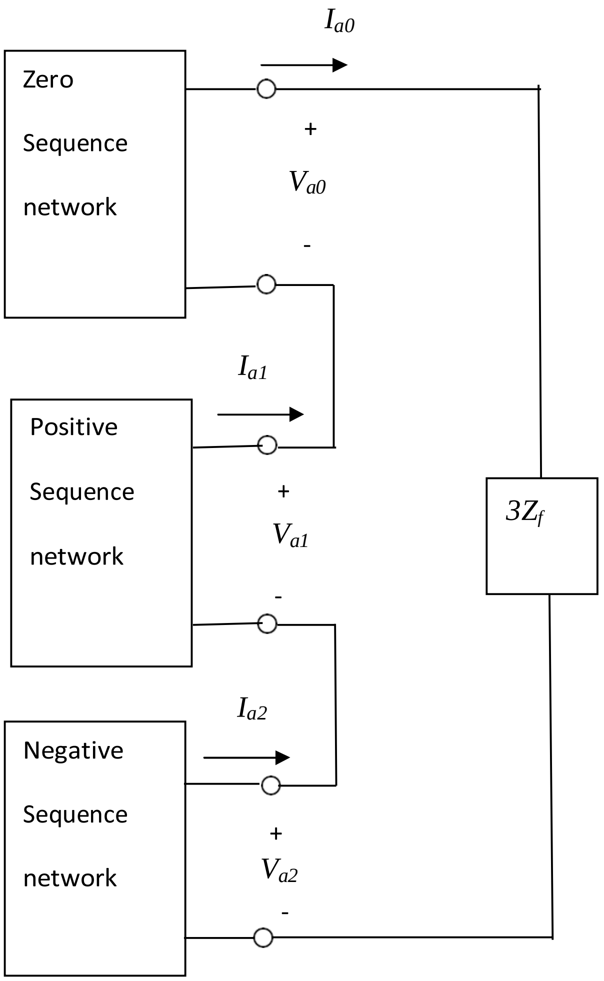


Fig.3.3 (a)

Equivalent sequence networks

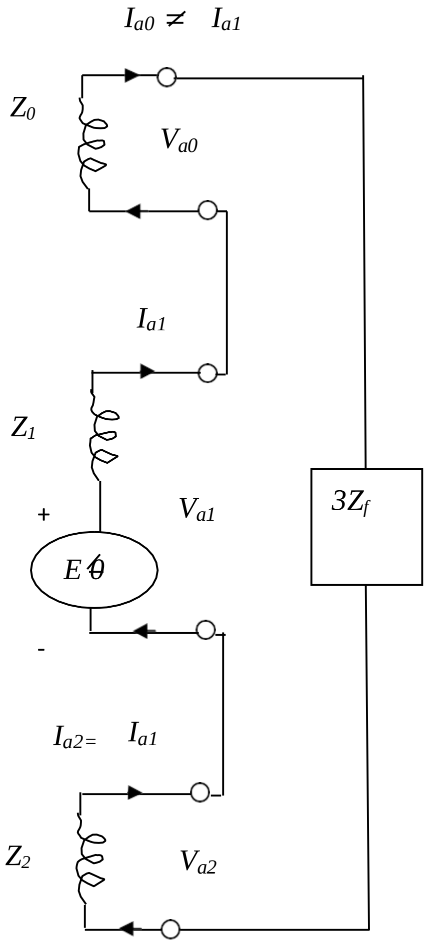


Fig.3.3 (b)

LINE TO LINE FAULT

The termination of the three- phase access port as in the fig.3.4 below simulates a line to line fault through a fault impedance Z_f .

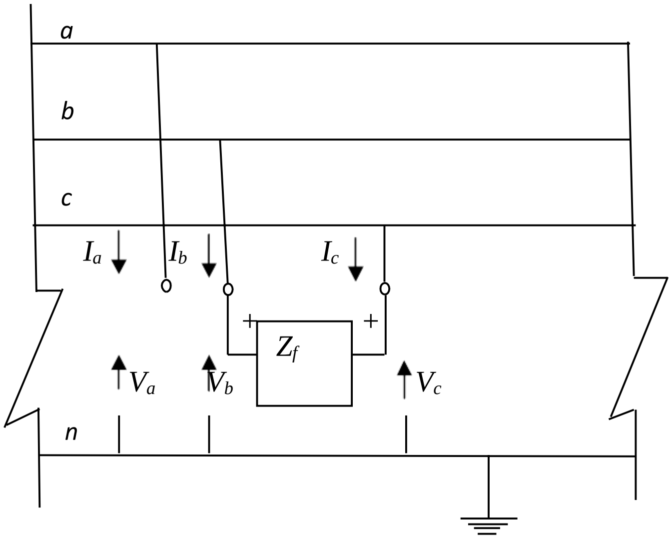


Fig. 3.4

The terminal conditions at the fault point give the following equations,

$I_a = 0$

$I_b = -I_c$

$V_b = V_c + Z_f I_b$

$I_b = -I_c = I_{a0} + a^2 I_{a1} + aI_{a2}$

Connection of sequence networks for a line to line fault and its simplified equivalent circuit are shown in the fig.3.5 (a) and fig.(b) below.

General sequence networks

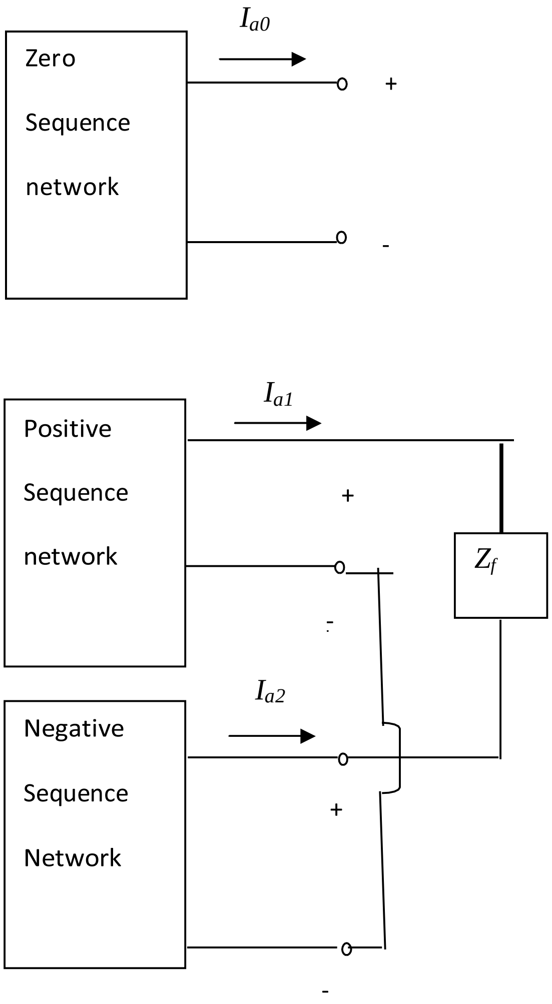


Fig. 3.5 (a)

Equivalent sequence networks

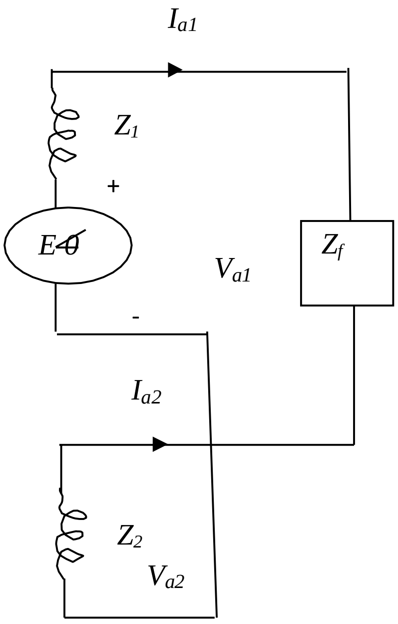


Fig.3.5 (b)

DOUBLE LINE TO GROUND FAULT

The termination of the three- phase access port as shown in fig.3.6 simulates a double line to ground fault through fault impedance Z_f.

The terminal conditions at the fault point give the following equations,

$I_a = 0$

$V_b = V_c = (I_b + I_c)\, Z_f$

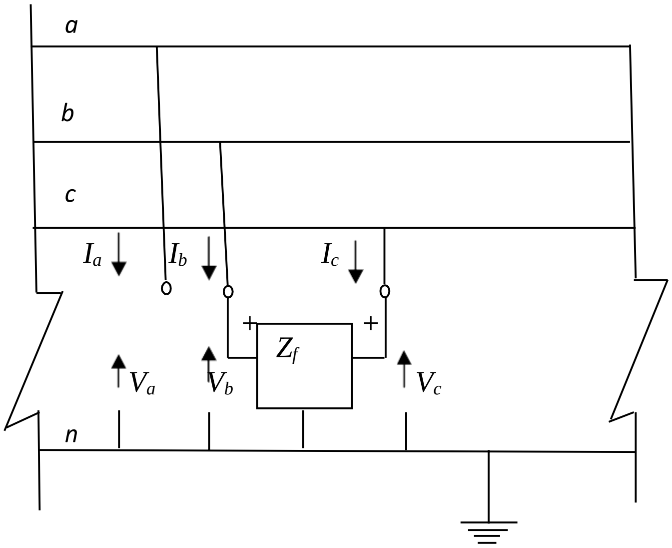


Fig. 3.6

The sequence networks and the equivalent circuit are shown by the Fig.3.7 (a) and Fig. 3.7 (b) below

General sequencenetworks

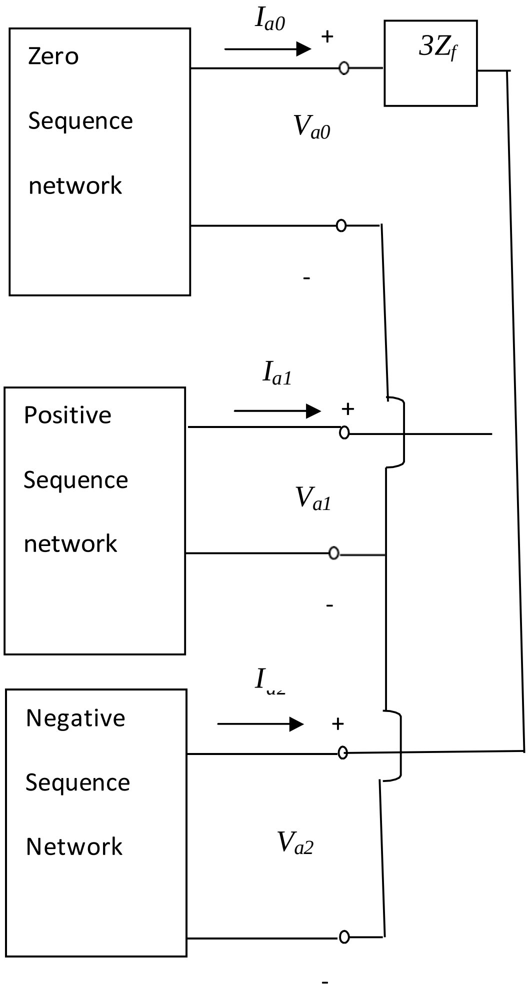


Fig. 3.7(a)

Equivalent sequence networks

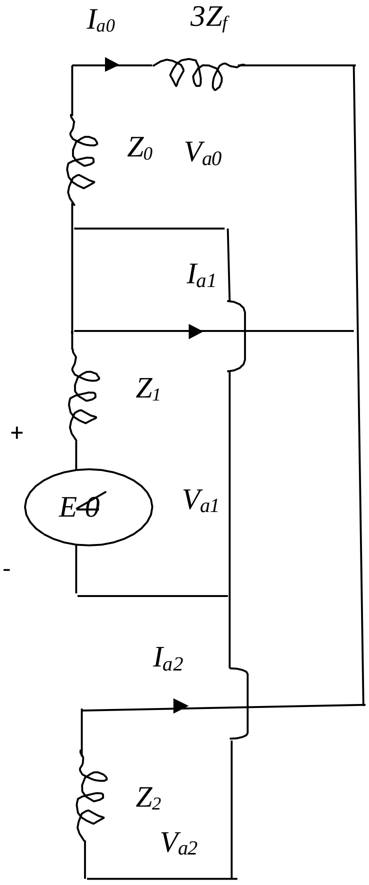


Fig.3.7 (b)

BALANCED THREE PHASE FAULT

This type of fault occurs infrequently, as for example, when a line, which has been made safe for maintenance by clamping all the three phases to earth, is accidentally made alive or when, due to slow fault clearance, an earth fault spreads across to the other two phases or when a mechanical excavator cuts quickly through a whole cable.

It is an important type of fault in that it results in an easy calculation and generally, a pessimistic answer.

The circuit breaker rated MVA breaking capacity is based on 3- phase fault MVA. Since circuit breakers are manufactured in preferred standard sizes e.g. 250, 500, 750 MVA high precision is not necessary when calculating the 3- phase fault level at a point in a power system.

The system impedances are also never known accurately in three phase faults.

UNIT-IV

LOAD FLOW STUDIES-1

REVIEW OF NUMERICAL SOLUTION OF EQUATIONS

The numerical analysis involving the solution of algebraic simultaneous equations forms the basis for solution of the performance equations in computer aided electrical power system analyses, such as during linear graph analysis, load flow analysis (nonlinear equations), transient stability studies (differential equations), etc. Hence, it is necessary to review the general forms of the various solution methods with respect to all forms of equations, as under:

1. Solution Linear equations:
* **Direct methods**:
- Cramer‟s (Determinant) Method,
- Gauss Elimination Method (only for smaller systems),
- LU Factorization (more preferred method), etc.

*** Iterative methods**:
- Gauss Method
- Gauss-Siedel Method (for diagonally dominant systems)

3. Solution of Nonlinear equations:
Iterative methods only:
- Gauss-Siedel Method (for smaller systems)
- Newton-Raphson Method (if corrections for variables are small)

4. Solution of differential equations:
Iterative methods only:
- Euler and Modified Euler method,
- RK IV-order method,
- Milne‟s predictor-corrector method, etc.

It is to be observed that the nonlinear and differential equations can be solved only by the iterative methods. The iterative methods are characterized by the various performance features as under:
_ Selection of initial solution/ estimates
_ Determination of fresh/ new estimates during each iteration
_ Selection of number of iterations as per tolerance limit
_ Time per iteration and total time of solution as per the solution method selected
_ Convergence and divergence criteria of the iterative solution
_ Choice of the Acceleration factor of convergence, etc.

A comparison of the above solution methods is as under:
In general, the direct methods yield exact or accurate solutions. However, they are suited for only the smaller systems, since otherwise, in large systems, the possible round-off errors make the solution process inaccurate. The iterative methods are more useful when the diagonal elements of the coefficient matrix are large in comparison with the offdiagonal elements. The round-off errors in these methods are corrected at the successive steps of the iterative process.The Newton-Raphson method is very much useful for solution of non –linear equations, if all the values of the corrections for the unknowns are very small in magnitude and the initial values of unknowns are selected to be reasonably closer to the exact solution.

LOAD FLOW STUDIES

Introduction: Load flow studies are important in planning and designing future expansion of power systems. The study gives steady state solutions of the voltages at all the buses, for a particular load condition. Different steady state solutions can be obtained, for different operating conditions, to help in planning, design and operation of the power system. Generally, load flow studies are limited to the transmission system, which involves bulk power transmission. The load at the buses is assumed to be known. Load flow studies throw light on some of the important aspects of the system operation, such as: violation of voltage magnitudes at the buses, overloading of lines, overloading of generators, stability margin reduction, indicated by power angle differences between buseslinked by a line, effect of contingencies like line voltages, emergency shutdown of generators, etc. Load flow studies are required for deciding the economic operation of the power system. They are also required in transient stability studies. Hence, load flow studies play a vital role in power system studies. Thus the load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions. At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle. Because of the nonlinearity of the algebraic equations, describing the given power system, their solutions are obviously, based on the iterative methods only. The constraints placed on the load flow solutions could be:
_ The Kirchhoff‟s relations holding good,
_ Capability limits of reactive power sources,
_ Tap-setting range of tap-changing transformers,
_ Specified power interchange between interconnected systems,
_ Selection of initial values, acceleration factor, convergence limit, etc.

Classification of buses for LFA: Different types of buses are present based on the specified and unspecified variables at a given bus as presented in the table below:

Table 1. Classification of buses for LFA

Sl. No.	Bus Types	Specified Variables	Unspecified variables	Remarks
1	Slack/ Swing Bus	$\lvert V\rvert$, δ	P_G, Q_G	$\lvert V\rvert$, δ: are assumed if not specified as 1.0 and 0^0
2	Generator/ Machine/ PV Bus	P_G, $\lvert V\rvert$	Q_G, δ	A generator is present at the machine bus
3	Load/ PQ Bus	P_G, Q_G	$\lvert V\rvert$, δ	About 80% buses are of PQ type
4	Voltage Controlled Bus	P_G,Q_G, $\lvert V\rvert$	δ, a	'a' is the % tap change in tap-changing transformer

Importance of swing bus: The slack or swing bus is usually a PV-bus with the largest capacity generator of the given system connected to it. The generator at the swing bus supplies the power difference between the "specified power into the system at the other buses" and the "total system output plus losses". Thus swing bus is needed to supply the additional real and reactive power to meet the losses. Both the magnitude and phase angle of voltage are specified at the swing bus, or otherwise, they are assumed to be equal to 1.0 p.u. and 00 , as per flat-start procedure of iterative
solutions. The real and reactive powers at the swing bus are found by the computer routine as part of the load flow solution process. It is to be noted that the source at the swing bus is a perfect one, called the swing machine, or slack machine. It is voltage regulated, i.e., the magnitude of voltage fixed. The phase angle is the system reference phase and hence is fixed. The generator at the swing bus has a torque angle and excitation which vary or swing as the demand changes. This variation is such as to produce fixed voltage.

Importance of YBUS based LFA:

The majority of load flow programs employ methods using the bus admittance matrix, as this method is found to be more economical. The bus admittance matrix plays a very important role in load flow analysis. It is a complex, square and symmetric matrix and hence only n(n+1)/2 elements of YBUS need to be stored for a n-bus system. Further, in the YBUS matrix, Yij = 0, if an incident element is not present in the system connectingthe buses „i‟ and „j‟. since in a large power system, each bus is connected only to a fewer buses through an incident element, (about 6-8), the coefficient matrix, YBUS of such systems would be highly sparse, i.e., it will have many zero valued elements in it. This is defined by the sparsity of the matrix, as under:

$$\text{Percentage sparsity of a given matrix of } n^{th} \text{ order:} = \frac{\text{Total no. of zero valued elements of } Y_{BUS}}{\text{Total no. of entries of } Y_{BUS}}$$

$$\mathbf{S} = (\mathbf{Z} / \mathbf{n}^2) \times \mathbf{100}\ \% \qquad \mathbf{(1)}$$

The percentage sparsity of Y_{BUS}, in practice, could be as high as 80-90%, especially
for very large, practical power systems. This sparsity feature of YBUS is extensively used in reducing the load flow calculations and in minimizing the memory required to store the

coefficient matrices. This is due to the fact that only the non-zero elements YBUS can be stored during the computer based implementation of the schemes, by adopting the suitable optimal storage schemes. While YBUS is thus highly sparse, it‟s inverse, ZBUS, the bus impedance matrix is not so. It is a FULL matrix, unless the optimal bus ordering schemes are followed before proceeding for load flow analysis.

THE LOAD FLOW PROBLEM

Here, the analysis is restricted to a balanced three-phase power system, so that the analysis can be carried out on a single phase basis. The per unit quantities are used for allquantities. The first step in the analysis is the formulation of suitable equations for the power flows in the system. The power system is a large interconnected system, where various buses are connected by transmission lines. At any bus, complex power is injected into the bus by the generators and complex power is drawn by the loads. Of course at any bus, either one of them may not be present. The power is transported from one bus to othervia the transmission lines. At any bus *i*, the complex power Si (injected), shown in figure 1, is defined as

$$S_i = S_{Gi} - S_{Di} \qquad \textbf{(2)}$$

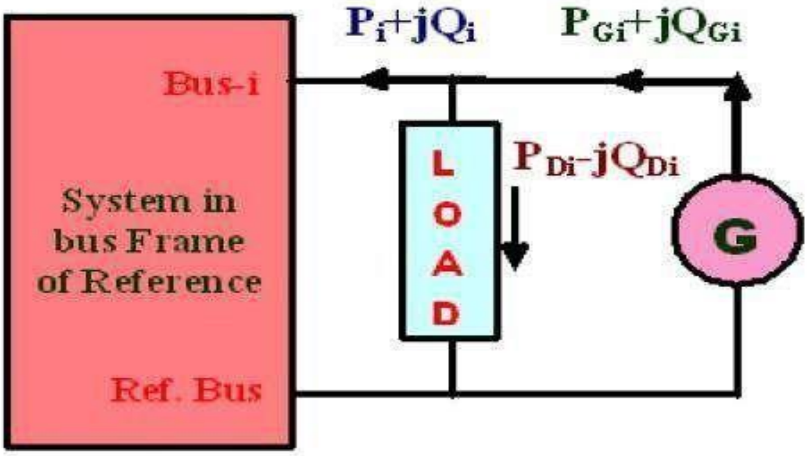


Fig.1 power flows at a bus-i

where Si = net complex power injected into bus *i*, SGi = complex power injected by the generator at bus *i*, and SDi = complex power drawn by the load at bus *i*. According to conservation of complex power, at any bus i, the complex power injected into the bus must be equal to the sum of complex power flows out of the bus via the transmission lines. Hence,

Si = _Sij " i = 1, 2, ...n
(3)

where Sij is the sum over all lines connected to the bus and *n* is the number of buses in the system (excluding the ground). The bus current injected at the bus-i is defined as

Ii = IGi – IDi " i = 1, 2, ………..n **(4)** where IGi is the current injected by the generator at the bus and IDi is the current drawn by the load (demand) at that bus. In the bus frame of reference

IBUS = YBUS VBUS
(5)

where

$$I_{BUS} = \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_n \end{bmatrix}$$ is the vector of currents injected at the buses,

Y_{BUS} is the bus admittance matrix, and

$$V_{BUS} = \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix}$$ is the vector of complex bus voltages.

Equation (5) can be considered as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \qquad \forall \ i = 1, 2, \ldots\ldots\ldots\ldots n \tag{6}$$

The complex power S_i is given by

$$\begin{aligned} S_i &= V_i \, I_i^* \\ &= V_i \left(\sum_{j=1}^{n} Y_{ij} \, V_j \right)^* \\ &= V_i \left(\sum_{j=1}^{n} Y_{ij}^* V_j^* \right) \end{aligned} \tag{7}$$

Let $$V_i \underline{\underline{\Delta}} |V_i| \angle \delta_i = |V_i| (\cos \delta_i + j \sin \delta_i)$$

$$\delta_{ij} = \delta_i - \delta_j$$

$$Y_{ij} = G_{ij} + jB_{ij}$$

Hence from (7), we get,

$$\mathrm{S_i} = \sum_{j=1}^{n} |V_i|\,|V_j|\,(\cos\delta_{ij} + j\sin\delta_{ij})\,(G_{ij} - j\,B_{ij}) \qquad \mathbf{(8)}$$

Separating real and imaginary parts in (8) we obtain,

$$\mathrm{P_i} = \sum_{j=1}^{n} |V_i|\,|V_j|\,(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}) \qquad \mathbf{(9)}$$

$$\mathrm{Q_i} = \sum_{j=1}^{n} |V_i|\,|V_j|\,(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}) \qquad \mathbf{(10)}$$

An alternate form of $\mathrm{P_i}$ and $\mathrm{Q_i}$ can be obtained by representing $\mathrm{Y_{ik}}$ also in polar form as

$$\mathrm{Y_{ij}} = |Y_{ij}|\,\angle\theta_{ij} \qquad \mathbf{(11)}$$

Again, we get from (7),

$$\mathrm{S_i} = |V_i|\angle\delta_i \sum_{j=1}^{n} |Y_{ij}|\,\angle-\theta_{ij}\,|V_j|\,\angle-\delta_j \qquad \mathbf{(12)}$$

The real part of (12) gives $\mathrm{P_i}$.

$$P_i = |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\cos(-\theta_{ij} + \delta_i - \delta_j)$$

$$= |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\cos-(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$P_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,|Y_{ij}|\,\cos(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\ \mathrm{i} = 1, 2, \ldots\ldots\ldots\mathrm{n}, \qquad \mathbf{(13)}$$

Similarly, $\mathrm{Q_i}$ is imaginary part of (12) and is given by

$$Q_i = |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\sin-(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$Q_i = -\sum_{j=1}^{n} |V_i|\,|V_j|\,|Y_{ij}|\,\sin(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\ \mathrm{i} = 1, 2, \ldots\ldots\ldots\mathrm{n} \qquad \mathbf{(14)}$$

Equations (9)-(10) and (13)-(14) are the „power flow equations" or the „load flow equations" in two alternative forms, corresponding to the n-bus system, where each bus-*i* is characterized by four variables, Pi, Qi, |Vi|, and di. Thus a total of 4n variables are involved in these equations. The load flow equations can be solved for any 2n unknowns, if the other 2n variables are specified. This establishes the need for classification of buses of the system for load flow analysis into: PV bus, PQ bus, etc.

DATA FOR LOAD FLOW

Irrespective of the method used for the solution, the data required is common for any load flow. All data is normally in pu. The bus admittance matrix is formulated from these data. The various data required are as under:

System data: It includes: number of buses-*n*, number of PV buses, number of loads, number of transmission lines, number of transformers, number of shunt elements, the slack bus number, voltage magnitude of slack bus (angle is generally taken as 0o), tolerance limit, base MVA, and maximum permissible number of iterations.

Generator bus data: For every PV bus *i*, the data required includes the bus number, active power generation PGi, the specified voltage magnitude *i sp V* , , minimum reactive power limit Qi,min, and maximum reactive power limit Qi,max.

Load data: For all loads the data required includes the the bus number, active power demand PDi, and the reactive power demand QDi.

Transmission line data: For every transmission line connected between buses *i* and *k* the data includes the starting bus number *i*, ending bus number *k*,.*r*esistance of the line, reactance of the line and the half line charging admittance.

Transformer data:
For every transformer connected between buses *i* and *k* the data to be given includes: the starting bus number *i*, ending bus number *k*, resistance of the transformer, reactance of the transformer, and the off nominal turns-ratio *a*.

Shunt element data: The data needed for the shunt element includes the bus number where element is connected, and the shunt admittance (Gsh + j Bsh).

GAUSS – SEIDEL (GS) METHOD

The GS method is an iterative algorithm for solving non linear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated till convergence is reached. The GS method applied to power flow problem is as discussed below.

Case (a): Systems with PQ buses only:

Initially assume all buses to be PQ type buses, except the slack bus. This means that (n–1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-i, given from (7), as:

$$S_i = V_i \left(\sum_{j=1}^{n} Y_{ij} V_j \right)^*$$

This can be written as

$$S_i^* = V_i^* \left(\sum_{j=1}^{n} Y_{ij} V_j \right) \qquad \textbf{(15)}$$

Since $S_i^* = P_i - jQ_i$, we get,

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^{n} Y_{ij} V_j$$

So that,

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \qquad \textbf{(16)}$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \right] \qquad \forall\ i = 2, 3, \ldots\ldots\ldots n \qquad \textbf{(17)}$$

Equation (17) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. Starting from an initial estimate of all bus voltages, in the RHS of (17) the most recent values of the bus voltages is substituted. One iteration of the method involves computation of all the bus voltages. In Gauss–Seidel method, the value of the updated voltages are used in the computation of subsequent voltages in the same iteration, thus speeding up convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value. Thus the algorithm for GS method is as under:

Algorithm for GS method

1. Prepare data for the given system as required.
2. Formulate the bus admittance matrix YBUS. This is generally done by the rule of inspection.
3. Assume initial voltages for all buses, 2,3,…n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be $1.0\angle 0^0$. This is normally refered as the ***flat start*** solution.
4. Update the voltages. In any (k +1)*st* iteration, from (17) the voltages are given by

$$V_i^{(k+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(k)})^*} - \sum_{j=1}^{i-1} Y_{ij} \mathrm{V}_j^{(k+1)} - \sum_{j=i+1}^{n} Y_{ij} \mathrm{V}_j^{(k)}\right] \forall \text{ i=2,3,…n} \qquad \textbf{(18)}$$

Here note that when computation is carried out for bus-i, updated values are already available for buses 2,3….(i-1) in the current *(k+1)st* iteration. Hence these values are used. For buses *(i+1)…..n,* values from previous, *kth* iteration are used.

$$\left|\Delta V_i^{(k+1)}\right| = \left|V_i^{(k+1)} - V_i^{(k)}\right| < \varepsilon \qquad \forall \text{ i = 2,3,…n} \qquad \textbf{(19)}$$

Where,e is the tolerance value. Generally it is customary to use a value of 0.0001 pu. Compute slack bus power after voltages have converged using (15) [assuming bus 1 is slack bus].

$$S_1^* = \mathrm{P}_1 - \mathrm{jQ}_1 = V_1^*\left(\sum_{j=1}^{n} Y_{1j} V_j\right) \qquad \textbf{(20)}$$

7. Compute all line flows.
8. The complex power loss in the line is given by Sik + Ski. The total loss in the system is calculated by summing the loss over all the lines.

Case (b): Systems with PV buses also present:
At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of Qi to be used in (18). From (15) we have

$$\mathrm{Q_i} = -\,\mathrm{Im}\left\{V_i^* \sum_{j=1}^{n} Y_{ij}\, V_j\right\}$$

Where *Im* stands for the imaginary part. At any $(k+1)^{st}$ iteration, at the PV bus-*i*,

$$Q_i^{(k+1)} = -\,\mathrm{Im}\left\{(V_i^{(k)})^* \sum_{j=1}^{i-1} Y_{ij}\, V_j^{(k+1)} + (V_i^{(k)})^* \sum_{j=i}^{n} Y_{ij}\, V_j^{(k)}\right\} \qquad \textbf{(21)}$$

The steps for i^{th} PV bus are as follows:

1. Compute $Q_i^{(k+1)}$ using (21)
2. Calculate $\mathrm{V_i}$ using (18) with $\mathrm{Q_i} = Q_i^{(k+1)}$
3. Since $|V_i|$ is specified at the PV bus, the magnitude of $\mathrm{V_i}$ obtained in step 2

has to be modified and set to the specified value $|V_{i,sp}|$. Therefore,

$$V_i^{(k+1)} = |V_{i,sp}| \angle \delta_i^{(k+1)} \qquad \textbf{(22)}$$

The voltage computation for PQ buses does not change.

Case (c): Systems with PV buses with reactive power generation limits specified:

In the previous algorithm if the Q limit at the voltage controlled bus is violated during any iteration, i.e $(k+1)$ $i\ Q$ computed using (21) is either less than Qi, min or greater than Qi,max, it means that the voltage cannot be maintained at the specified value due to lackof reactive power support. This bus is then treated as a PQ bus in the *(k+1)st* iteration and the voltage is calculated with the value of Qi set as follows:

If $\mathrm{Q_i} < \mathrm{Q_{i,min}}$ If $\mathrm{Q_i} > \mathrm{Q_{i,max}}$

Then $\mathrm{Q_i} = \mathrm{Q_{i,min.}}$ Then $\mathrm{Q_i} = \mathrm{Q_{i,max.}}$

(23)

If in the subsequent iteration, if Qi falls within the limits, then the bus can be switched back to PV status.

Acceleration of convergence

It is found that in GS method of load flow, the number of iterations increase with increase in the size of the system. The number of iterations required can be reduced if the correction in voltage at each bus is accelerated, by multiplying with a constant α, called the acceleration factor. In the (k+1)st iteration we can let

$$V_i^{(k+1)}(accelerate\ d) = V_i^{(k)} + \alpha\left(V_i^{(k+1)} - V_i^{(k)}\right) \qquad \textbf{(24)}$$

where α is a real number. When α =1, the value of $(k+1)$ is the computed value. If 1<α <2 then the value computed is extrapolated. Generally _ is taken between 1.2 to 1.6, for GS load flow procedure. At PQ buses (pure load buses) if the voltage magnitude violates the limit, it simply means that the specified reactive power demand cannot be supplied, with the voltage maintained within acceptable limits.

Examples on GS load flow analysis:

Example-1: *Obtain the voltage at bus 2 for the simple system shown in Fig 2, using the Gauss–Seidel method, if* $V_1 = 1\angle 0^0$ *pu.*

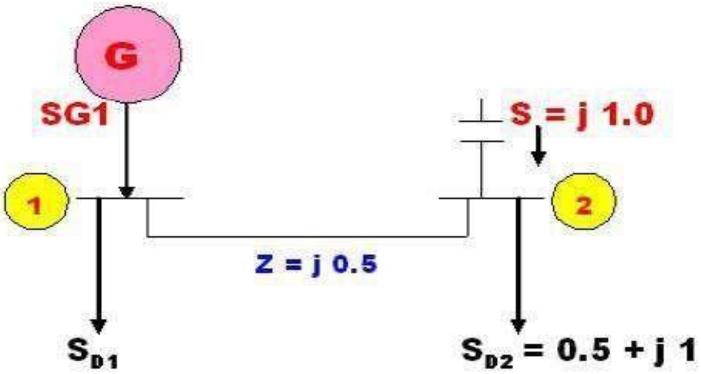


Fig : System of Example 1

Solution:

Here the capacitor at bus 2, injects a reactive power of 1.0 pu. The complex power injection at bus 2 is

$$S_2 = j1.0 - (0.5 + j\,1.0) = -\,0.5 \text{ pu.}$$

$$V_1 = 1\angle 0^0$$

$$Y_{BUS} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$V_2^{(k+1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{\left(V_2^{(k)}\right)^*} - Y_{21}\,V_1\right]$$

Since V_1 is specified it is a constant through all the iterations. Let the initial voltage at bus 2, $V_2^0 = 1 + j\,0.0 = 1\angle 0^0$ pu.

$$V_2^1 = \frac{1}{-j2}\left[\frac{-0.5}{1\angle 0^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 1.0 - j0.25 = 1.030776\ \angle -14.036^0$$

$$V_2^2 = \frac{1}{-j2}\left[\frac{-0.5}{1.030776\angle 14.036^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.94118 - j\,0.23529\ = 0.970145\ \angle -14.036^0$$

$$V_2^3 = \frac{1}{-j2}\left[\frac{-0.5}{0.970145\angle 14.036^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.9375 - j\,0.249999\ = 0.970261\ \angle -14.931^0$$

$$V_2^4 = \frac{1}{-j2}\left[\frac{-0.5}{0.970261\angle 14.931^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.933612 - j\,0.248963 = 0.966237\ \angle -14.931^0$$

$$V_2^5 = \frac{1}{-j2}\left[\frac{-0.5}{0.966237\angle 14.931^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.933335 - j\,0.25\ \ = 0.966237\ \angle -14.995^0$$

Since the difference in the voltage magnitudes is less than 10-6 pu, the iterations can be stopped. To compute line flow

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{1\angle 0^0 - 0.966237\angle -14.995^0}{j0.5}$$

$$= 0.517472 \angle -14.931^0$$

$$S_{12} = V_1 I_{12}^* = 1\angle 0^0 \times 0.517472 \angle 14.931^0$$

$$= 0.5 + \mathrm{j}\ 0.133329 \text{ pu}$$

$$I_{21} = \frac{V_2 - V_1}{Z_{12}} = \frac{0.966237\angle -14.995^0 - 1\angle 0^0}{j0.5}$$

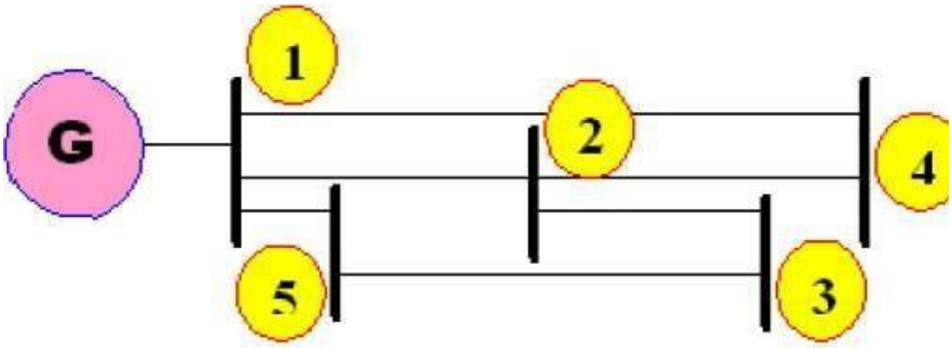
$$= 0.517472 \angle -194.93^0$$

$$S_{21} = V_2 I_{21}^* = -0.5 + \mathrm{j}\ 0.0 \text{ pu}$$

The total loss in the line is given by S12 + S21 = j 0.133329 pu Obviously, it is observed that there is no real power loss, since the line has no resistance.

Example-2:

For the power system shown in fig. below, with the data as given in tables below, obtain the bus voltages at the end of first iteration, by applying GS method.



Power System of Example 2

Line data of example 2

SB	EB	R (pu)	X (pu)	$\frac{B_C}{2}$
1	2	0.10	0.40	-
1	4	0.15	0.60	-
1	5	0.05	0.20	-
2	3	0.05	0.20	-
2	4	0.10	0.40	-
3	5	0.05	0.20	-

Bus data of example 2

Bus No.	P_G (pu)	Q_G (pu)	P_D (pu)	Q_D (pu)	$\lvert V_{SP}\rvert$ (pu)	δ
1	-	-	-	-	1.02	0°
2	-	-	0.60	0.30	-	-
3	1.0	-	-	-	1.04	-
4	-	-	0.40	0.10	-	-
5	-	-	0.60	0.20	-	-

Solution: In this example, we have,

- Bus 1 is slack bus, Bus 2, 4, 5 are PQ buses, and Bus 3 is PV bus
- The lines do not have half line charging admittances

  $P_2 + jQ_2 = P_{G2} + jQ_{G2} - (P_{D2} + jQ_{D2}) = -0.6 - j0.3$

$P_3 + jQ_3 = P_{G3} + jQ_{G3} - (P_{D3} + jQ_{D3}) = 1.0 + jQ_{G3}$

Similarly $P_4 + jQ_4 = -0.4 - j0.1$, $P_5 + jQ_5 = -0.6 - j0.2$

The Y_{bus} formed by the rule of inspection is given by:

Y_{bus} =

2.15685 -j8.62744	-0.58823 +j2.35294	0.0+j0.0	-0.39215 +j1.56862	-1.17647 +j4.70588
-0.58823 +j2.35294	2.35293 -j9.41176	-1.17647 +j4.70588	-0.58823 +j2.35294	0.0+j0.0
0.0+j0.0	-1.17647 +j4.70588	2.35294 -j9.41176	0.0+j0.0	-1.17647 +j4.70588
-0.39215 +j1.56862	-0.58823 +j2.35294	0.0+j0.0	0.98038 -j3.92156	0.0+j0.0
-1.17647 +j4.70588	0.0+j0.0	-1.17647 +j4.70588	0.0+j0.0	2.35294 -j9.41176

The voltages at all PQ buses are assumed to be equal to 1+j0.0 pu. The slack bus voltage is taken to be V_1^0 = 1.02+j0.0 in all iterations.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21}\,V_1^o - Y_{23}\,V_3^0 - Y_{24}\,V_4^0 - Y_{25}\,V_5^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{-0.6 + j0.3}{1.0 - j0.0} - \{(-0.58823 + j2.35294)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times 1.04\angle 0^o\} - \{(-0.58823 + j2.35294)\times 1.0\angle 0^0\}\right]$$

$$= 0.98140\angle -3.0665^o = 0.97999 - j0.0525$$

Bus 3 is a PV bus. Hence, we must first calculate Q_3. This can be done as under:

$$Q_3 = |V_3|\,|V_1|\,(G_{31}\sin\delta_{31} - B_{31}\cos\delta_{31}) + |V_3|\,|V_2|\,(G_{32}\sin\delta_{32} - B_{32}\cos\delta_{32})$$

$$+ |V_3|^2\,(G_{33}\sin\delta_{33} - B_{33}\cos\delta_{33}) + |V_3|\,|V_4|\,(G_{34}\sin\delta_{34} - B_{34}\cos\delta_{34})$$

$$+ |V_3|\,|V_5|\,(G_{35}\sin\delta_{35} - B_{35}\cos\delta_{35})$$

We note that $\delta_1 = 0^o$; $\delta_2 = -3.0665^o$; $\delta_3 = 0^o$; $\delta_4 = 0^o$ and $\delta_5 = 0^o$

$\therefore\ \delta_{31} = \delta_{33} = \delta_{34} = \delta_{35} = 0^o$ ($\delta_{ik} = \delta_i - \delta_k$); $\delta_{32} = 3.0665^o$

$Q_3 = 1.04\ [1.02\ (0.0+j0.0) + 0.9814\ \{-1.17647 \times \sin(3.0665^o) - 4.70588 \times\cos(3.0665^o)\} + 1.04\{-9.41176 \times\cos(0^o)\} + 1.0\ \{0.0 + j0.0\} + 1.0\{-4.70588\times\cos(0^0)\}]$

$= 1.04\ [-4.6735 + 9.78823 - 4.70588] = 0.425204$ pu.

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}\,V_1^o - Y_{32}\,V_2^1 - Y_{34}\,V_4^0 - Y_{35}\,V_5^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{1.0 - j0.425204}{1.04 - j0.0} - \{(-1.7647 + j4.70588)\times(0.98140\angle-3.0665^o)\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times(1\angle 0^o)\}\right]$$

$$= 1.05569\angle 3.077^o = 1.0541 + j0.05666 \text{ pu.}$$

Since it is a PV bus, the voltage magnitude is adjusted to specified value and V_3^1 is computed as: $V_3^1 = 1.04\angle 3.077^0$ pu

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}V_1^o - Y_{42}V_2^1 - Y_{43}V_3^1 - Y_{45}V_5^0\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{-0.4 + j0.1}{1.0 - j0.0} - \{(-0.39215 + j1.56862)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-0.58823 + j2.35294)\times(0.98140\angle-3.0665^o)\}\right]$$

$$= \frac{0.45293 - j3.8366}{0.98038 - j3.92156} = 0.955715\angle-7.303^o \text{ pu} = 0.94796 - j0.12149$$

$$V_5^1 = \frac{1}{Y_{55}}\left[\frac{P_5 - jQ_5}{V_5^{o*}} - Y_{51}V_1^o - Y_{52}V_2^1 - Y_{53}V_3^1 - Y_{54}V_4^1\right]$$

$$= \frac{1}{Y_{55}}\left[\frac{-0.6 + j0.2}{1.0 - j0.0} - \{(-1.17647 + j4.70588)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times 1.04\angle 3.077^o\}\right]$$

$$= 0.994618\angle-1.56^o = 0.994249 - j0.027$$

Thus at end of 1st iteration, we have,

$V_1 = 1.02\angle 0^0$ pu $\qquad V_2 = 0.98140\ \angle-3.066^0$ pu

$V_3 = 1.04\angle 3.077^0$ pu $\qquad V_4 = 0.955715\angle-7.303^0$ pu

and $\qquad V_5 = 0.994618\angle-1.56^0$ pu

Example-3:

Obtain the load flow solution at the end of first iteration of the system with data as given below. The solution is to be obtained for the following cases

(i) All buses except bus 1 are PQ Buses

(ii) Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

(iii) Bus 2 is PV bus, with voltage magnitude specified as 1.04 and 0.25_Q2_1.0 pu.

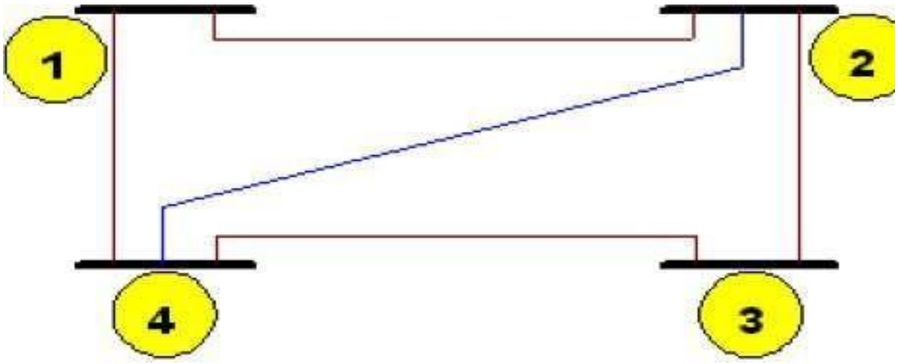
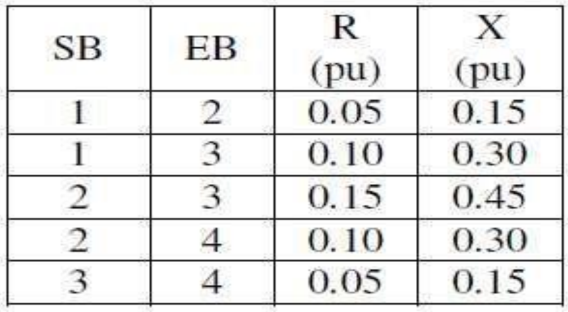


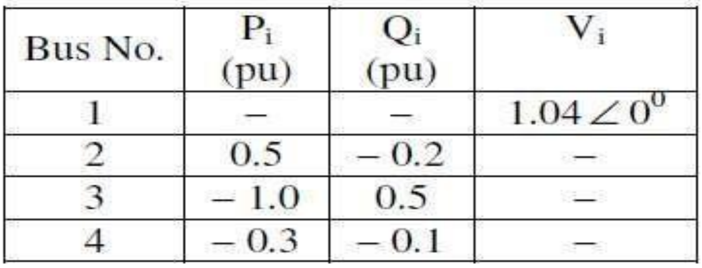
Fig. System for Example 3

Table: Line data of example 3

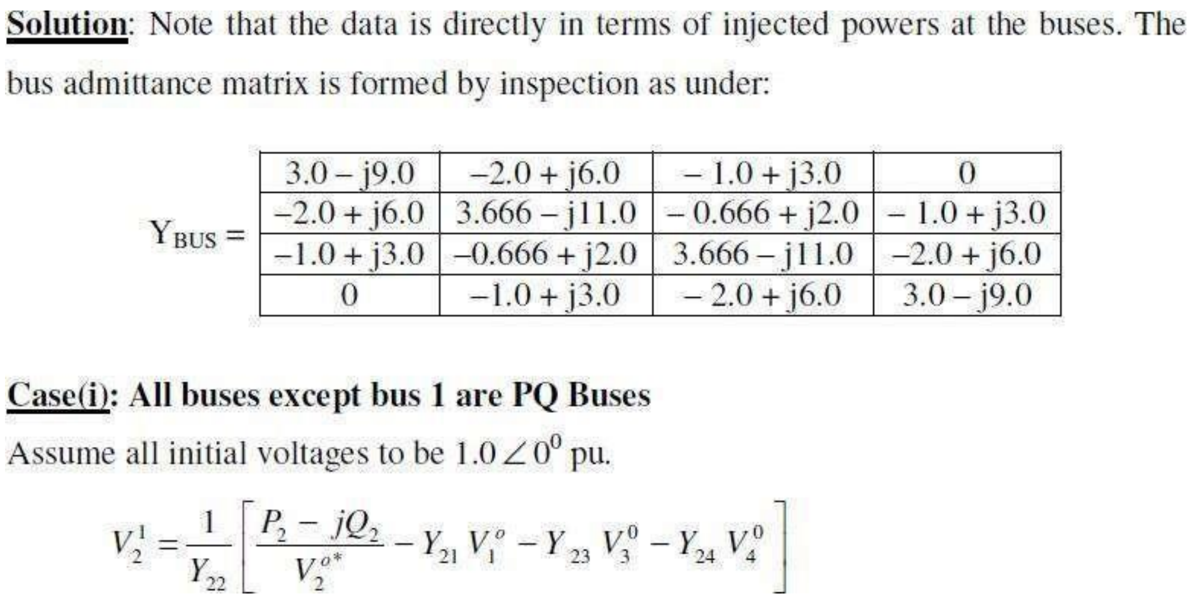


SB	EB	R (pu)	X (pu)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

Table: Bus data of example 3



Bus No.	P_i (pu)	Q_i (pu)	V_i
1	–	–	$1.04\angle 0^0$
2	0.5	– 0.2	–
3	– 1.0	0.5	–
4	– 0.3	– 0.1	–



Solution: Note that the data is directly in terms of injected powers at the buses. The bus admittance matrix is formed by inspection as under:

$$Y_{BUS} = \begin{bmatrix} 3.0 - j9.0 & -2.0 + j6.0 & -1.0 + j3.0 & 0 \\ -2.0 + j6.0 & 3.666 - j11.0 & -0.666 + j2.0 & -1.0 + j3.0 \\ -1.0 + j3.0 & -0.666 + j2.0 & 3.666 - j11.0 & -2.0 + j6.0 \\ 0 & -1.0 + j3.0 & -2.0 + j6.0 & 3.0 - j9.0 \end{bmatrix}$$

Case(i): All buses except bus 1 are PQ Buses

Assume all initial voltages to be $1.0 \angle 0^0$ pu.

$$V_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21} V_1^o - Y_{23} V_3^0 - Y_{24} V_4^0 \right]$$

$$= \frac{1}{Y_{22}}\left[\frac{0.5 + j0.2}{1.0 - j0.0} - \{(-2.0 + j6.0)\times(1.04\angle 0^o)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.0\angle 0^o)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.02014\angle 2.605^{\circ}$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}V_1^o - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.02014\angle 2.605^o)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.03108\angle -4.831^{\circ}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}V_1^o - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.02014\angle 2.605^o)\}\right.$$

$$\left. - \{(-2.0 + j6.0)\times(1.03108\angle -4.831^o)\}\right]$$

$$= 1.02467\angle -0.51^{\circ}$$

Hence

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.02014\ \angle 2.605^0$ pu

$V_3^1 = 1.03108\ \angle -4.831^0$ pu $\qquad V_4^1 = 1.02467\angle -0.51^0$ pu

Case(ii): Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

We first compute Q_2.

$$Q_2 = |V_2|\big[|V_1|(G_{21}\sin\delta_{21} - B_{21}\cos\delta_{21}) + |V_2|(G_{22}\sin\delta_{22} - B_{22}\cos\delta_{22})$$
$$+ |V_3|(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23}) + |V_4|(G_{24}\sin\delta_{24} - B_{24}\cos\delta_{24})\big]$$

$$= 1.04\ [1.04\ \{-6.0\} + 1.04\ \{11.0\} + 1.0\{-2.0\} + 1.0\ \{-3.0\} = 0.208 \text{ pu.}$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{0.5 - j0.208}{1.04\angle 0^0} - \{(-2.0 + j6.0)\times(1.04\angle 0^o)\}\right.$$
$$\left. - \{(-0.666 + j2.0)\times(1.0\angle 0^o)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^0)\}\right]$$
$$= 1.051288 + j0.033883$$

The voltage magnitude is adjusted to 1.04. Hence $V_2^1 = 1.04\angle 1.846^0$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0\ \angle 0^0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\}\right.$$
$$\left. - \{(-0.666 + j2.0)\times(1.04\angle 1.846^o)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^0)\}\right]$$
$$= 1.035587\angle -4.951^o \text{ pu.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 1.846^o)\}\right.$$
$$\left. - \{(-2.0 + j6.0)\times(1.035587\angle -4.951^o)\}\right]$$
$$= 0.9985\angle -0.178^o$$

Hence at end of 1st iteration we have:

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.04\ \angle 1.846^0$ pu

$V_3^1 = 1.035587\ \angle -4.951^0$ pu $\qquad V_4^1 = 0.9985\angle -0.178^0$ pu

Case (iii): Bus 2 is PV bus, with voltage magnitude specified as 1.04 & $0.25 \leq Q_2 \leq 1$ pu. If $0.25 \leq Q_2 \leq 1.0$ pu then the computed value of $Q_2 = 0.208$ is less than the lower limit. Hence, Q_2 is set equal to 0.25 pu. Iterations are carried out with this value of Q_2. The voltage magnitude at bus 2 can no longer be maintained at 1.04. Hence, there is no necessity to adjust for the voltage magnitude. Proceeding as before we obtain at the end of first iteration,

$$V_1^1 = 1.04 \angle 0^0 \text{ pu} \qquad V_2^1 = 1.05645 \angle 1.849^0 \text{ pu}$$

$$V_3^1 = 1.038546 \angle -4.933^0 \text{ pu} \qquad V_4^1 = 1.081446 \angle 4.896^0 \text{ pu}$$

Limitations of GS load flow analysis

GS method is very useful for very small systems. It is easily adoptable, it can be generalized and it is very efficient for systems having less number of buses. However, GS LFA fails to converge in systems with one or more of the features as under:

- Systems having large number of radial lines
- Systems with short and long lines terminating on the same bus
- Systems having negative values of transfer admittances
- Systems with heavily loaded lines, etc.

GS method successfully converges in the absence of the above problems. However, convergence also depends on various other set of factors such as: selection of slack bus, initial solution, acceleration factor, tolerance limit, level of accuracy of results needed, type and quality of computer/ software used, etc.

UNIT–V

LOAD FLOW STUDIES II:

NEWTON –RAPHSON METHOD

Newton-Raphson (NR) method is used to solve a system of non-linear algebraic equations of the form f(x) =0. Consider a set of n non-linear algebraic equations given by

$$f_i(x_1, x_2 \ldots\ldots x_n) = 0 \qquad i = 1,2\ldots\ldots n \qquad \textbf{(25)}$$

Let $x_1^{\,0}, x_2^{\,0} \ldots\ldots\ldots x_n^{\,0}$, be the initial guess of unknown variables and $\Delta x_1^{\,0}, \Delta x_2^{\,0} \ldots\ldots \Delta x_n^{\,0}$ be the respective corrections. Therefore,

$$f_i(x_1^{\,0} + \Delta x_1^{\,0}, x_2^{\,0} + \Delta x_2^{\,0} \ldots\ldots\ldots x_n^{\,0} + \Delta x_n^{\,0}) = 0 \qquad i = 1,2\ldots\ldots n \qquad \textbf{(26)}$$

The above equation can be expanded using Taylor's series to give

$$f_i(x_1^{\,0}, x_2^{\,0} \ldots\ldots x_n^{\,0}) + \left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^{\,0} + \left(\frac{\partial f_i}{\partial x_2}\right)\Delta x_2^{\,0} + \ldots\ldots + \left(\frac{\partial f_i}{\partial x_n}\right)\Delta x_n^{\,0}\right]$$

$$+ \text{ Higher order terms} = 0 \qquad \forall \ i = 1,2\ldots\ldots n \qquad \textbf{(27)}$$

Where, $\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0, \ldots\ldots\ldots \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the partial derivatives of f_i with respect to $x_1, x_2 \ldots\ldots x_n$ respectively, evaluated at ($x_1^{\,0}, x_2^{\,0} \ldots\ldots\ldots x_0^{\,n}$). If the higher order terms are neglected, then (27) can be written in matrix form as

$$\begin{bmatrix} f_1^{\,0} \\ f_2^{\,0} \\ \cdot \\ \cdot \\ \cdot \\ f_n^{\,0} \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \left(\frac{\partial f_n}{\partial x_1}\right) & \left(\frac{\partial f_n}{\partial x_2}\right) & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_n}{\partial x_n}\right) \end{bmatrix} \begin{bmatrix} \Delta x_1^{\,0} \\ \Delta x_2^{\,0} \\ \cdot \\ \cdot \\ \cdot \\ \Delta x_n^{\,0} \end{bmatrix} = 0 \qquad \textbf{(28)}$$

In vector form (28) can be written as

$$F^0 + J^0 \Delta X^0 = 0$$

Or $$F^0 = -J^0 \Delta X^0$$

Or $$\Delta X^0 = -[J^0]^{-1} F^0 \qquad \textbf{(29)}$$

And $$X^1 = X^0 + \Delta X^0 \qquad \textbf{(30)}$$

Here, the matrix [J] is called the **Jacobian** matrix. The vector of unknown variables is updated using (30). The process is continued till the difference between two successive iterations is less than the tolerance value.

NR method for load flow solution in polar coordinates

In application of the NR method, we have to first bring the equations to be solved, to the form $f_i(x_1, x_2, \ldots x_n) = 0$, where $x_1, x_2, \ldots x_n$ are the unknown variables to be determined. Let us assume that the power system has n_1 PV buses and n_2 PQ buses. In polar coordinates the unknown variables to be determined are:

(i) δ_i, the angle of the complex bus voltage at bus *i*, at all the PV and PQ buses. This gives us $n_1 + n_2$ unknown variables to be determined.

(ii) $|V_i|$, the voltage magnitude of bus *i*, at all the PQ buses. This gives us n_2 unknown variables to be determined.

Therefore, the total number of unknown variables to be computed is: $n_1 + 2n_2$, for which we need $n_1 + 2n_2$ consistent equations to be solved. The equations are given by,

$$\Delta P_i = P_{i,sp} - P_{i,cal} = 0 \quad \textbf{(31)}$$

$$\Delta Q_i = Q_{i,sp} - Q_{i,cal} = 0 \quad \textbf{(32)}$$

Where $P_{i,sp}$ = Specified active power at bus *i*

$Q_{i,sp}$ = Specified reactive power at bus *i*

$P_{i,cal}$ = Calculated value of active power using voltage estimates.

$Q_{i,cal}$ = Calculated value of reactive power using voltage estimates

ΔP = Active power residue

ΔQ = Reactive power residue

The real power is specified at all the PV and PQ buses. Hence (31) is to be solved at all PV and PQ buses leading to $n_1 + n_2$ equations. Similarly the reactive power is specified at all the PQ buses. Hence, (32) is to be solved at all PQ buses leading to n_2 equations.

We thus have $n_1 + 2n_2$ equations to be solved for $n_1 + 2n_2$ unknowns. (31) and (32) are of the form F(x) = 0. Thus NR method can be applied to solve them. Equations (31) and (32) can be written in the form of (30) as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \tag{33}$$

Where J_1, J_2, J_3, J_4 are the negated partial derivatives of ΔP and ΔQ with respect to corresponding δ and $|V|$. The negated partial derivative of ΔP, is same as the partial derivative of P_{cal}, since P_{sp} is a constant. The various computations involved are discussed in detail next.

Computation of P_{cal} and Q_{cal}:

The real and reactive powers can be computed from the load flow equations as:

$$P_{i,Cal} = P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$= G_{ii}|V_i|^2 + \sum_{\substack{k=1\\k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \tag{34}$$

$$Q_{i,Cal} = Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$= -B_{ii}|V_i|^2 + \sum_{\substack{k=1\\k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) \tag{35}$$

The powers are computed at any $(r+1)^{st}$ iteration by using the voltages available from previous iteration. The elements of the Jacobian are found using the above equations as:

Elements of J_1

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1\\k\neq i}}^{n} |V_i||V_k|\{G_{ik}(-\sin\delta_{ik}) + B_{ik}\cos\delta_{ik}\}$$

$$= -Q_i - B_{ii}|V_i|^2$$

$$\frac{\partial P_i}{\partial \delta_k} = |V_i||V_k|(G_{ik}(-\sin\delta_{ik})(-1) + B_{ik}(\cos\delta_{ik})(-1))$$

Elements of J$_3$

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i - G_{ii}|V_i|^2$$

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J$_2$

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = 2|V_i|^2 G_{ii} + |V_i|\sum_{\substack{k=1 \\ k\neq i}}^{n} |V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i + |V_i|^2 G$$

$$\frac{\partial P_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J$_4$

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = -2|V_i|^2 B_{ii} + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) = Q_i - |V_i|^2$$

$$\frac{\partial Q_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

Thus, the linearized form of the equation could be considered agai

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \dfrac{\Delta|V|}{|V|} \end{bmatrix}$$

The elements are summarized below:

(i) $H_{ii} = \dfrac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii}|V_i|^2$

(ii) $H_{ik} = \dfrac{\partial P_i}{\partial \delta_k} = a_k f_i - b_k e_i = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$

(iii) $N_{ii} = \dfrac{\partial P_i}{\partial |V_i|}|V_i| = P_i + G_{ii}|V_i|^2$

(iv) $N_{ik} = \dfrac{\partial P_i}{\partial |V_k|}|V_k| = a_k e_i + b_k f_i = |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$

(v) $M_{ii} = \dfrac{\partial Q_i}{\partial \delta_i} = P_i - G_{ii}|V_i|^2$

DECOUPLED LOAD FLOW

In the NR method, the inverse of the Jacobian has to be computed at every iteration. When solving large interconnected power systems, alternative solution methods are possible, taking into account certain observations made of practical systems. These are,

- Change in voltage magnitude $|V_i|$ at a bus primarily affects the flow of reactive power Q in the lines and leaves the real power P unchanged. This observation implies that $\frac{\partial Q_i}{\partial |V_j|}$ is much larger than $\frac{\partial P_i}{\partial |V_j|}$. Hence, in the Jacobian, the elements of the sub-matrix $[N]$, which contains terms that are partial derivatives of real power with respect to voltage magnitudes can be made zero.

the

- Change in voltage phase angle at a bus, primarily affects the real power flow P over the lines and the flow of Q is relatively unchanged. This observation implies that $\frac{\partial P_i}{\partial \delta_j}$ is much larger than $\frac{\partial Q_i}{\partial \delta_j}$. Hence, in the Jacobian the elements of the sub-matrix $[M]$, which contains terms that are partial derivatives of reactive power with respect to voltage phase angles can be made zero.

, L

These observations reduce the NRLF linearised form of equation to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \frac{\Delta|V|}{|V|} \end{bmatrix} \qquad \textbf{(37)}$$

From (37) it is obvious that the voltage angle corrections $\Delta\delta$ are obtained using real power residues ΔP and the voltage magnitude corrections $|\Delta V|$ are obtained from reactive power residues ΔQ. This equation can be solved through two alternate strategies as under:

Strategy-1

(i) Calculate $\Delta P^{(r)}, \Delta Q^{(r)}$ and $J^{(r)}$

(ii) Compute $\begin{bmatrix} \Delta\delta^{(r)} \\ \dfrac{\Delta\left|V^{(r)}\right|}{\left|V^{(r)}\right|} \end{bmatrix} = \left[J^{(r)}\right]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$

(iii) Update δ and $|V|$.

(iv) Go to step (i) and iterate till convergence is reached.

Strategy-2

(i) Compute $\Delta P^{(r)}$ and Sub-matrix $H^{(r)}$. From (37) find $\Delta\delta^{(r)} = \left[H^{(r)}\right]^{-1} \Delta P^{(r)}$

(ii) Up date δ using $\delta^{(r+1)} = \delta^{(r)} + \Delta\delta^{(r)}$.

(iii) Use $\delta^{(r+1)}$ to calculate $\Delta Q^{(r)}$ and $L^{(r)}$

(iv) Compute $\dfrac{\Delta\left|V^{(r)}\right|}{\left|V^{(r)}\right|} = \left[L^{(r)}\right]^{-1} \Delta Q^{(r)}$

(v)Update, $\left|V^{(r+1)}\right| = \left|V^{(r)}\right| + \left|\Delta V^{(r)}\right|$

(vi) Go to step (i) and iterate till convergence is reached.

In the first strategy, the variables are solved simultaneously. In the second strategy the iteration is conducted by first solving for $\Delta\delta$ and using updated values of δ to calculate $\Delta|V|$. Hence, the second strategy results in faster convergence, compared to the first strategy.

FAST DECOUPLED LOAD FLOW

If the coefficient matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of fast decoupled load Flow (FDLF). Here, certain assumptions are made based on the observations of practical power systems as under:

- $B_{ij} >> G_{ij}$ (Since the X/R ratio of transmission lines is high in well designed systems)

- The voltage angle difference $(\delta_i - \delta_j)$ between two buses in the system is very small. This means $\cos(\delta_i - \delta_j) \cong 1$ and $\sin(\delta_i - \delta_j) = 0.0$
- $Q_i << B_{ii}|V_i|^2$

With these assumptions the elements of the Jacobian become

$$H_{ik} = L_{ik} = -|V_i||V_k|B_{ik} \qquad (i \neq k)$$

$$H_{ii} = L_{ii} = -B_{ii}|V_i|^2$$

The matrix (37) reduces to

$$[\Delta P] = \left[|V_i||V_j|B'_{ij}\right][\Delta\delta]$$

$$[\Delta Q] = \left[|V_i||V_j|B''_{ij}\right]\left[\frac{\Delta|V|}{|V|}\right] \qquad \textbf{(38)}$$

Where B'_{ij} and $B_{ij}^{''}$ are negative of the susceptances of respective elements of the bus admittance matrix. In (38) if we divide LHS and RHS by $|V_i|$ and assume $|V_j| \cong 1$, we get,

$$\left[\frac{\Delta P}{|V|}\right] = \left[B'_{ij}\right][\Delta\delta]$$

$$\left[\frac{\Delta Q}{|V|}\right] = \left[B''_{ij}\right]\left[\frac{\Delta|V|}{|V|}\right] \qquad \textbf{(39)}$$

Equations (39) constitute the Fast Decoupled load flow equations. Further simplification is possible by:

- Omitting effect of phase shifting transformers
- Setting off-nominal turns ratio of transformers to 1.0
- In forming B'_{ij}, omitting the effect of shunt reactors and capacitors which mainly affect reactive power
- Ignoring series resistance of lines in forming the Y_{bus}.

With these assumptions we obtain a loss-less network. In the FDLF method, the matrices $[B']$ and $[B'']$ are constants and need to be inverted only once at the beginning of the iterations.

REPRESENTATION OF TAP CHANGING TRANSFORMERS

Consider a tap changing transformer represented by its admittance connected in series with an ideal autotransformer as shown (a= turns ratio of transformer)

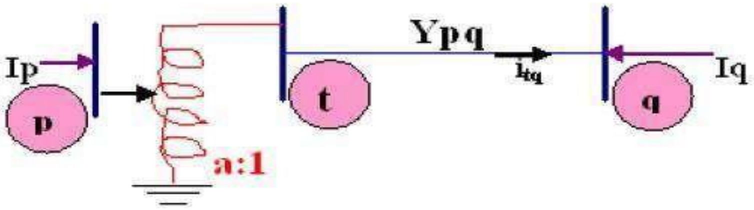


Fig. 2. Equivalent circuit of a tap setting transformer

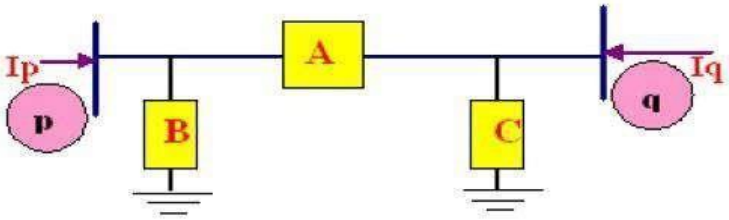


Fig. 3. π-Equivalent circuit of Fig.2 above.

By equating the bus currents in both the mutually equivalent circuits as above, it can be shown that the π-equivalent circuit parameters are given by the expressions as under:

(i) Fixed tap setting transformers (on no load)

A = Ypq/ a

B = 1/a (1/a -1) Ypq

C = (1-1/a) Ypq

(i) Tap changing under load (TCUL) transformers (on load)

A = Ypq

B = (1/a -1) (1/a + 1 – Eq/Ep) Ypq

C = (1-1/a) (Ep/Eq) Ypq

Thus, here, in the case of TCUL transformers, the shunt admittance values are observed to be a function of the bus voltages.

COMPARISON OF LOAD FLOW METHODS

The comparison of the methods should take into account the computing time required for preparation of data in proper format and data processing, programming ease, storage requirements, computation time per iteration, number of iterations, ease and time required for modifying network data when operating conditions change, etc. Since all the methods presented are in the bus frame of reference in admittance form, the data preparation is same for all the methods and the bus admittance matrix can be formed using a simple algorithm, by the rule of inspection. Due to simplicity of the equations, Gauss-Seidel method is relatively easy to program. Programming of NR method is more involved and becomes more complicated if the buses are randomly numbered. It is easier to program, if the PV buses are ordered in sequence and PQ buses are also ordered in sequence.

The storage requirements are more for the NR method, since the Jacobian elements have to be stored. The memory is further increased for NR method using rectangular coordinates. The storage requirement can be drastically reduced by using sparse matrix techniques, since both the admittance matrix and the Jacobian are sparse matrices. The time taken for a single iteration depends on the number of arithmetic and logical operations required to be performed in a full iteration. The Gauss –Seidel method requires the fewest number of operations to complete iteration. In the NR method, the computation of the Jacobian is necessary in every iteration. Further, the inverse of the Jacobian also has to be computed. Hence, the time per iteration is larger than in the GS method and is roughly about 7 times that of the GS method, in large systems, as depicted graphically in figure below. Computation time can be reduced if the Jacobian is updated once in two or three iterations. In FDLF method, the Jacobian is constant and needs to be computed only once. In both NR and FDLF methods, the time per iteration increases directly as the number of buses.

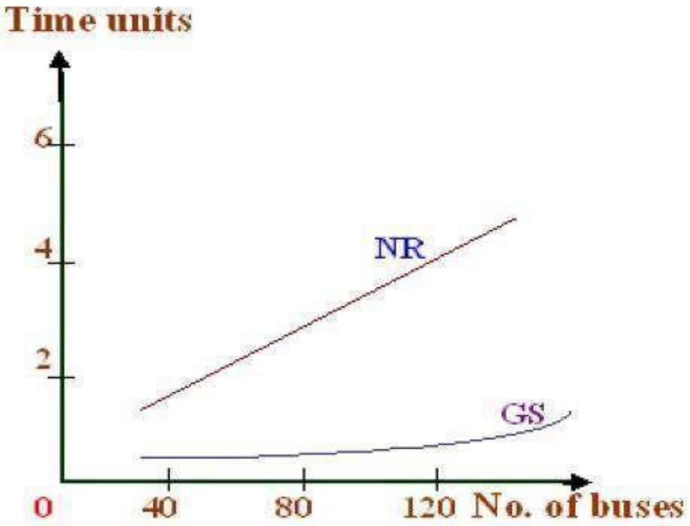


Figure 4. Time per Iteration in GS and NR methods

The number of iterations is determined by the convergence characteristic of the method. The GS method exhibits a linear convergence characteristic as compared to the NR method which has a quadratic convergence. Hence, the GS method requires more number of iterations to get a converged solution as compared to the NR method. In the GS method, the number of iterations increases directly as the size of the system increases. In contrast, the number of iterations is relatively constant in NR and FDLF methods. They require about 5-8 iterations for convergence in large systems. A significant increase in rate of convergence can be obtained in the GS method if an acceleration factor is used. All these variations are shown graphically in figure below. The number of iterations also depends on the required accuracy of the solution. Generally, a voltage tolerance of 0.0001 pu is used to obtain acceptable accuracy and the real power mismatch and reactive power mismatch can be taken as 0.001 pu. Due to these reasons, the NR method is faster and more reliable for large systems. The convergence of FDLF method is geometric and its speed is nearly 4-5 times that of NR method.

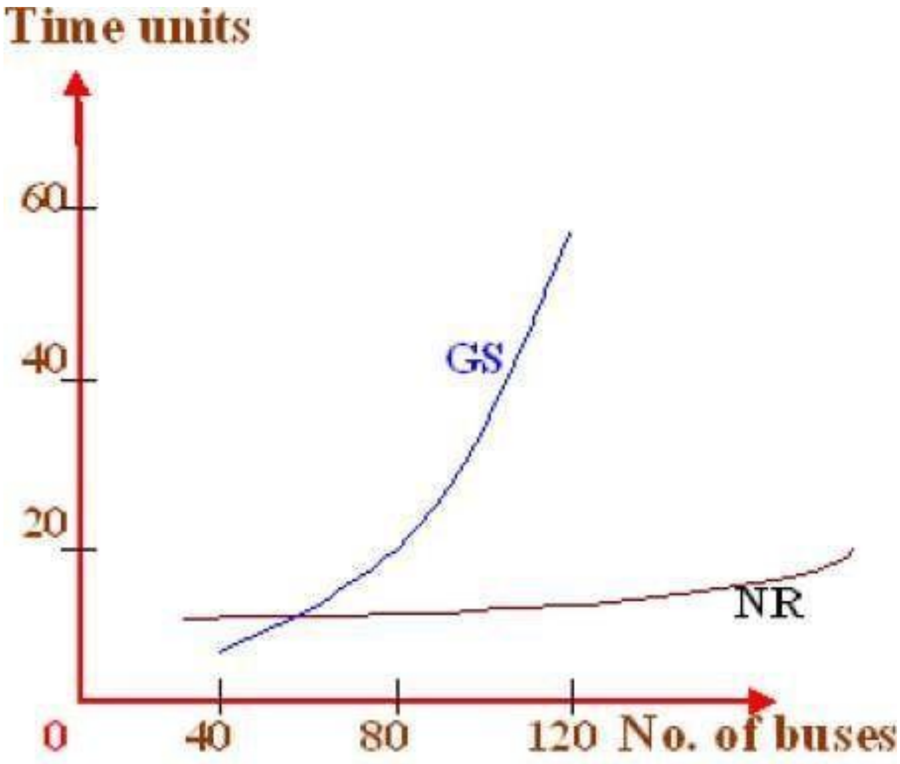


Figure 5. Total time of Iteration in GS and NR methods

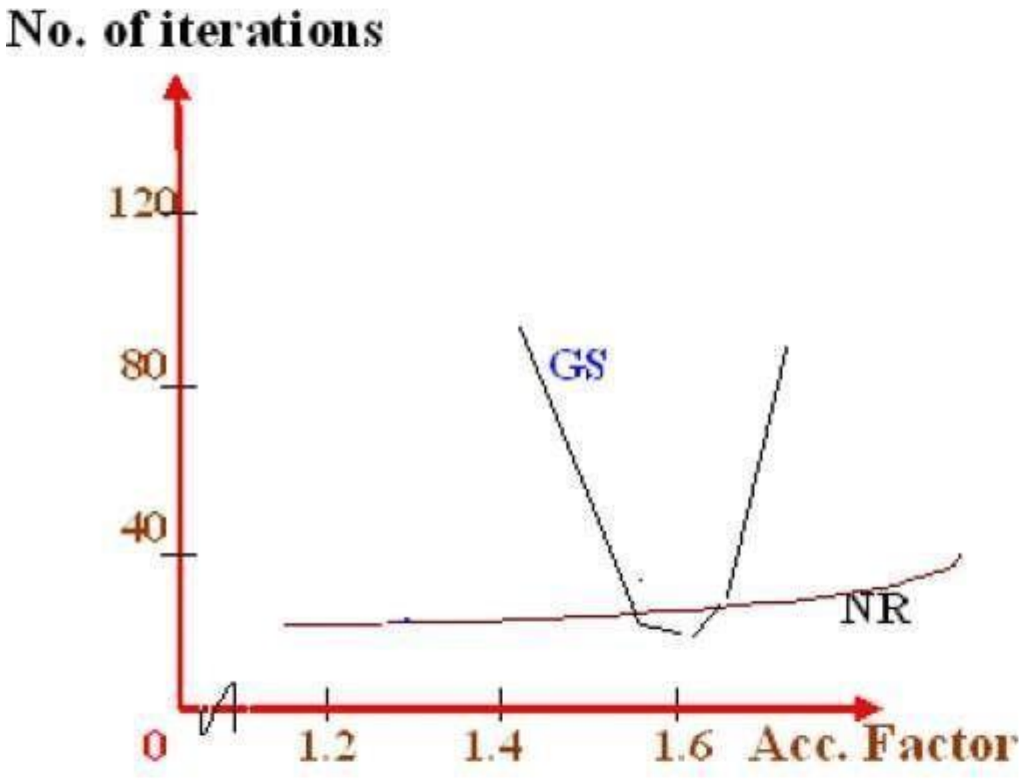


Figure 6. Influence of acceleration factor on load flow methods

FINAL WORD

In this chapter, the load flow problem, also called as the power flow problem, has been considered in detail. The load flow solution gives the complex voltages at all the buses and the complex power flows in the lines. Though, algorithms are available using the impedance form of the equations, the sparsity of the bus admittance matrix and the ease of building the bus admittance matrix, have made algorithms using the admittance form of equations more popular. The most popular methods are the Gauss-Seidel method, the Newton-Raphson method and the Fast Decoupled Load Flow method. These methods have been discussed in detail with illustrative examples. In smaller systems, the ease of programming and the memory requirements, make GS method attractive. However, the computation time increases with increase in the size of the system. Hence, in large systems NR and FDLF methods are more popular. There is a trade off between various requirements like speed, storage, reliability, computation time, convergence characteristics etc. No single method has all the desirable features. However, NR method is most popular because of its versatility, reliability and accuracy.